UNIVERSITY OF CONNECTICUT

Fall 2018 - Math 2410 **Name (Print):** ____________________

Exam 2 - October 30

Time Limit: 75 Minutes

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	10	
2	16	
3	15	
4	23	
5	18	
6	18	
Total:	100	

Do not write in the table to the right.

1. (10 points) Consider the initial value problem

$$y' = 2x + y^2 \quad \text{with} \quad y(1) = 2.$$

Use Euler's method to obtain an approximation of $y(2)$ using the step size $h = .5$.

2. (16 points) Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between the temperature of the object itself and the temperature of its surroundings.

   A cup with water at 45 °C is placed in the cooler held at 5 °C. If after 2 minutes the water temperature is 25 °C, when will the water temperature be 15 °C?

3. Given that $y_1(x) = x^2 - 2x$ and $y_2(x) = xe^{-x}$ are both solutions of a certain second order homogeneous equation

$$y'' + p(x)y' + q(x)y = 0$$

for some nice $p(x)$ and $q(x)$.

Answer each of the following questions. Show your work in each part.

(a) (6 points) Determine the Wronskian of y_1 and y_2, $W(y_1, y_2)$.

(b) (3 points) Determine largest possible intervals for which y_1 and y_2 form a set of fundamental solutions of this equation.

(c) (6 points) True or false: $y_4(x) = 2x^2 - 4x + 2xe^{2-x}$ is also a solution. You must show your explanation.

4. Consider the given non-homogeneous differential equation

$$y'' - 4y' + 4y = 2x^2 + 1 + e^x.$$

   (a) (6 points) Find the complementary solution to the homogeneous DE

$$y'' - 4y' + 4y = 0.$$

   (b) (12 points) Using the method of undetermined coefficients find the particular solution to the non-homogeneous DE

$$y'' - 4y' + 4y = 2x^2 + 1 + e^x.$$

   (c) (5 points) First give the general solution of the DE and then find the solution satisfying the boundary conditions

$$y(0) = 3 \quad \text{and} \quad y(1) = 2e^2 + e + \frac{5}{2}.$$

5. (18 points) Find a differential equation that has

$$y(x) = c_1 e^{\frac{x}{2}} + c_2 x e^{\frac{x}{2}} + 12 + 4x^2 e^{\frac{x}{2}}$$

as its general solution.

6. (18 points) It is given that $y_1(x) = x$ and $y_2(x) = x^3$ are linearly independent solutions of

$$y'' - \frac{3}{x}y' + \frac{3}{x^2}y = 0 \quad \text{for } x > 0.$$

Using variations of parameter, find the general solution of the following non-homogeneous DE

$$y'' - \frac{3}{x}y' + \frac{3}{x^2}y = 4x\ln x.$$

(Hint: $\int 2x \ln x dx = x^2 \ln x - \frac{x^2}{2} + C$)