UCONN

UNIVERSITY OF CONNECTICUT

Fall 2018 - Math 2410
Exam 3 - December 6
Time Limit: 75 Minutes

Name (Print): ____________________

This exam contains 10 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	30	
2	16	
3	18	
4	18	
5	18	
6	0	
7	0	
Total:	100	

1. (a) (8 points) Find $\mathcal{L}\{f(t)\}$ where

$$f(t) = \begin{cases} e^{-t} & \text{when } 0 \leq t < 5, \\ -1 & \text{when } t \geq 5 \end{cases}$$

(b) (6 points) Find $\mathcal{L}\{3t^2 + e^2 \sin(2t)\}$.

(c) (8 points) Find the integral

$$\int_0^\infty e^{-(s+4)t}\cos(2t)dt \qquad \text{(assuming } s > 0\text{).}$$

(d) (8 points) Find $\mathcal{L}\{(16-t^2)\mathcal{U}(t-4)\}$ where $\mathcal{U}(t-a) = \begin{cases} 0, & 0 \le t < a, \\ 1, & t \ge a. \end{cases}$

2. (a) (8 points) Given

$$F(s) = \frac{3s - 3}{s^2 + 2s + 10}.$$

Find $\mathcal{L}^{-1}\{F(s)\}$.

(b) (8 points) Given

$$G(s) = \frac{4e^{-s}}{s^4 + 4s^2}.$$

Find $\mathcal{L}^{-1}\{G(s)\}$.

3. Suppose that $\mathcal{L}\{f(t)\} = F(s) = \frac{3s^2}{s^5+1}$ for some function $f(t)$ with given that $f(0) = 1$, $f'(0) = -6$, and $f''(0) = 1$.

   (a) (4 points) Find $\mathcal{L}\{tf(t)\}$.

   (b) (3 points) Find $\mathcal{L}\{f''(t))\}$.

(c) (5 points) Let $\mathcal{L}\{g(t)\} = \frac{s^5+1}{s^3}$. Find $f * g$.
(Hint: you may not need to use the integral definition of convolution).

(d) (6 points) (Recall that $\mathcal{L}\{f(t)\} = F(s) = \frac{3s^2}{s^5+1}$ for some function $f(t)$ with given that $f(0) = 1$, $f'(0) = -6$, and $f''(0) = 1$).
Let $g(t) = \mathcal{L}^{-1}\{e^{-6s}\frac{3s^2}{s^5+1}\}$. Find $g(6)$.

4. (18 points) Using Laplace transform method solve the following DE

$$y'(t) + 3\int_0^t y(t-\tau)\cos(\tau)d\tau = 0 \quad \text{with } y(0) = 1.$$

5. (18 points) Using Laplace transform method solve the following DE

$$y'' + 4y' + 4y = 2te^{-2t} \quad \text{with} \quad y(0) = 1 \quad \text{and} \quad y'(0) = -2.$$

6. **LRC-Series Circuits**. The charge on the capacitor is related to the current $i(t)$ by $i = dq/dt$ which satisfies

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t).$$

Let $L = 5\,\mathrm{h}$, $R = 20\Omega$, $C = \frac{1}{20}\,\mathrm{f}$, $E(t) = 4\,\mathrm{V}$, $q(0) = 0\,\mathrm{C}$, $i(0) = 5\,\mathrm{A}$.

(a) (5 points (bonus)) Find the charge $q(t)$.

7. (18 points (bonus)) Solve the following IVP

$$y'' + 2ty' - 4y = 1 \quad \text{with} \quad y(0) = 0 = y'(0).$$