UCONN
UNIVERSITY OF CONNECTICUT

Fall 2018 - Math 2410 **Name (Print):** ____________________
Practice Exam 1 - September 18
Time Limit: 75 Minutes

This exam contains 8 pages (including this cover page) and 8 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	10	
3	10	
4	10	
5	10	
6	10	
7	20	
8	10	
Total:	100	

1. For each part in (a) and (b), write down the letter corresponding to the equation on the list with the specified properties. Also answer parts (c), (d), and (e).

   A. $y^{(2410)} + 5y + y' = \cos(y)$.
   B. $\frac{dy}{dx} = \frac{6y}{x}$.
   C. $\frac{dy}{dx} = 2y^3 - 16$.
   D. $(y')^{2018} + 2y^{2410} = 0$.

   (a) (4 points) First order linear differential equation which is separable equation.

   (b) (4 points) First order autonomous differential equation.

   (c) (4 points) What is a suitable integrating factor that could be used to solve the linear differential equation you found in part (a)?

   (d) (4 points) Write down the order of the differential equations in (A) to (D).

   A. B. C. D.

   (e) (4 points) Test the DE from (A) to (D) if they are linear or nonlinear.

   A. B. C. D.

2. Consider the following differential equation

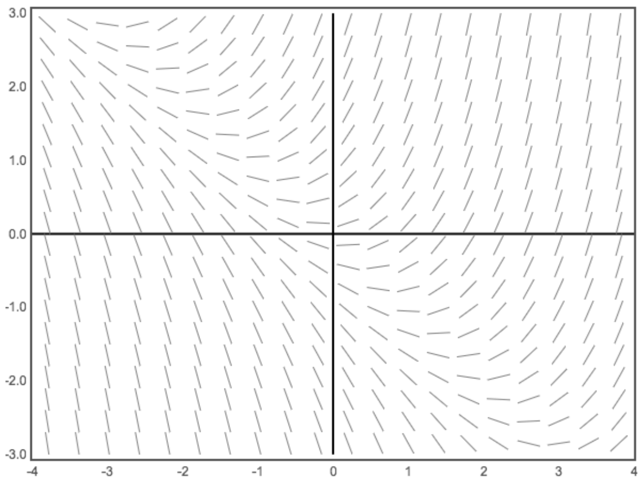
$$xy^2 + x^2 + (x^2y + y)y' = 0.$$

(a) (3 points) Is the differential equation exact?

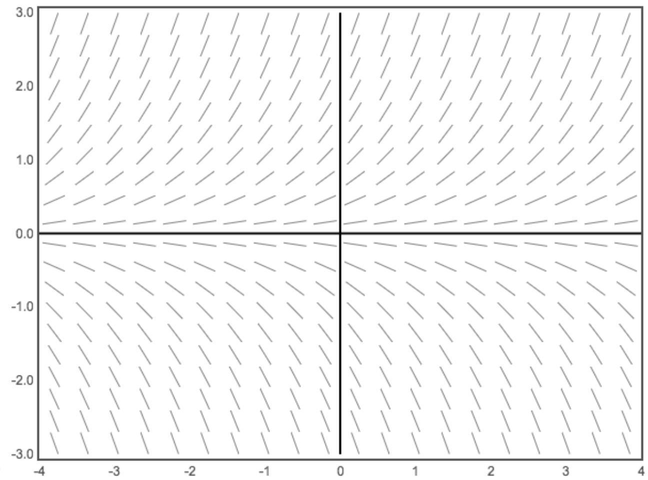
(b) (4 points) Find the 1-parameter family of solution of the differential equation (leave the solution as an implicit function).

(c) (3 points) Find the particular solution to the initial value problem $y(0) = 2$.

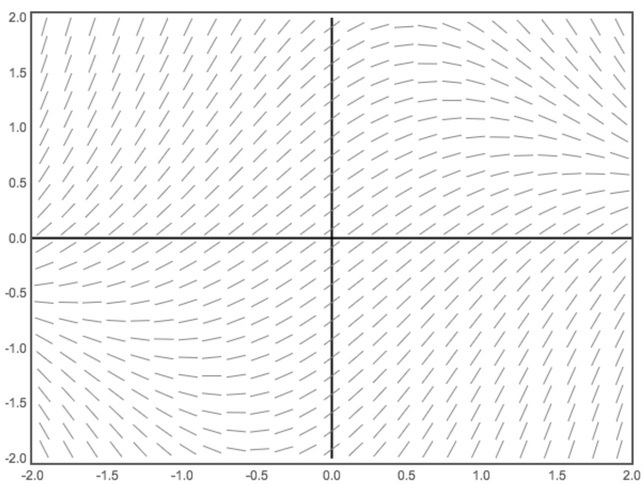
3. Consider the following direction fields (slope fields);



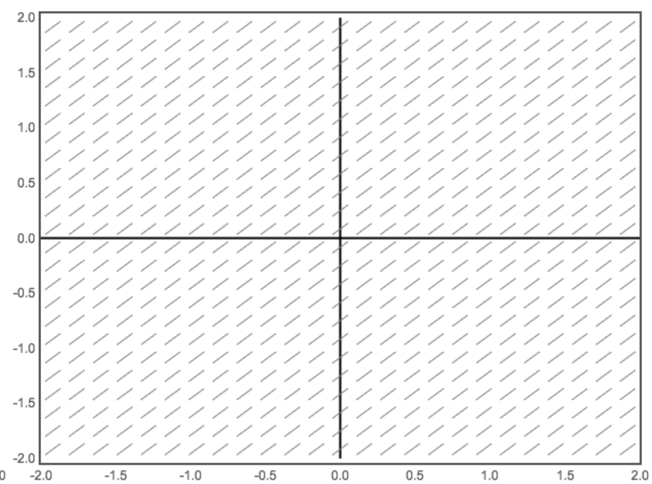
(a) ...



(b) ...



(c) ...



(d) ...

and the differential equations;

1. $\frac{dy}{dx} = y.$
2. $\frac{dy}{dx} = 1 - xy.$
3. $\frac{dy}{dx} = 1.$
4. $\frac{dy}{dx} = x + y.$

(a) (4 points) Match the given direction fields (a) to (d) and differential equations (1) to (4).

(b) (3 points) For each of the direction field (a) to (d), draw at least 3 solutions curves on the given graph.

(c) (3 points) For each of the direction field (a) to (d), draw the solution passing through $(-1, -1)$.

4. (10 points) Consider the initial value problem

$$(t^2 - 4)y' + \frac{t+2}{t}y = \frac{t^3}{t-5}, \quad y(4) = \frac{1}{2}.$$

Without solving the equation, what is the largest interval for t in which a unique solution is guaranteed to exist?

5. (10 points) Using the separation of variables method, solve the differential equation

$$y' + 2x(y+1) = 0 \text{ with } y(0) = 2.$$

6. Consider the following differential equation

$$2y' + y = e^x.$$

(a) (5 points) Find the 1-parameter family of solution of the differential equation. Write your solution in *explicit* form. (i.e., solve for y).

(b) (2 points) Using part (a), find the solution of the differential equation with the given initial value $y(0) = \alpha$.

(c) (3 points) For what value(s) of α, the solution you found in (b) remains finite as $x \to -\infty$?

7. Consider the autonomous equation

$$y' = y^2(3-y)(3+y).$$

(a) (4 points) Find all equilibrium solutions.

(b) (6 points) Classify the stability of each equilibrium solution as asymptotically stable(attractor), semi-stable, or unstable(repeller).

(c) (3 points) If $y(22/7) = \pi$, what is $\lim_{t\to\infty} y(t)$? (Hint: $\pi = 3.14159265...$)

(d) (3 points) If $y(2\pi) = -3$, what is $y(t)$?

(e) (4 points) If $y(4) = \lambda$. Find intervals for λ for which $\lim_{t\to\infty} y(t) = 0$.

8. (10 points) Solve the following Bernoulli equation

$$xy' + y + x^2y^2e^x = 0.$$

with using the substitution $u = y^{-1}$.