Fall 2018 - Math 2410
Practice Exam 2 - October 30
Time Limit: 75 Minutes

Name (Print): ____________________

This exam contains 8 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	12	
2	16	
3	15	
4	12	
5	15	
6	15	
7	15	
Total:	100	

1. (12 points) Consider the initial value problem

$$y' = 2x - 3y + 1 \quad \text{with} \quad y(1) = 5.$$

Use Euler's method to obtain an approximation of $y(1.2)$ using the step size $h = .1$.

2. Newtons law of cooling states that the temperature of an object changes at a rate proportional to the difference between the temperature of the object itself and the temperature of its surroundings.

   (a) (5 points) There is a cup of ice water in a room with ambient temperature T_s, which satisfies at time t, $T_s(t) = 70 + e^{-t}\sin(t)$. The initial temperature of the ice water is $30°F$. Assume the absolute value of the proportionality constant k is 1. Let $T(t)$ be the temperature of the ice water after time t. Write a differential equation with an initial value for T.

   (b) (6 points) What is the temperature of the ice water at time t?

   (c) (5 points) Determine $\lim_{t\to\infty} T(t)$. Show your work

3. (a) (4 points) Verify that $y_1(x) = e^{2x}$ and $y_2(x) = e^{5x}$ are the solutions to $y'' - 7y' + 10y = 0$ on $(-\infty, \infty)$.

(b) (4 points) Verify that $y_1(x) = e^{2x}$ and $y_2(x) = e^{5x}$ are linearly independent solutions of the above DE on $(-\infty, \infty)$.

(c) (4 points) Verify that $y_p(x) = 6e^x$ is a particular solution to the DE

$$y'' - 7y' + 10y = 24e^x \quad \text{on } (-\infty, \infty).$$

(d) (3 points) Combining your work (a)-(c), conclude that $y(x) = c_1e^{2x} + c_2e^{5x} + 6e^x$ is the general solution of the non-homogeneous DE

$$y'' - 7y' + 10y = 24e^x \quad \text{on } (-\infty, \infty).$$

4. (12 points) Find the general solution of the given higher order differential equation

$$y''' - 4y'' - 5y' = 0.$$

5. For linear differential equations of the form

$$x^2y'' + axy' + by = 0$$

we look for solution of the form $y(x) = x^r$.

(a) (6 points) Find two solutions of the form $y(x) = x^r$ for the DE

$$x^2y'' + xy' - 16y = 0.$$

(b) (3 points) Use part (a) and write down the general solution of the DE.

(c) (6 points) Using the general solution you found in part (a), find the particular solution satisfying the initial values

$$y(1) = 0 \quad \text{and} \quad y'(1) = 1.$$

6. (15 points) Find a differential equation that has

$$y(x) = c_1 e^{2x} \cos(3x) + c_2 e^{2x} \sin(3x) + \frac{1}{2} \sin(3x)$$

as its general solution.

7. (15 points) Find the solution to the initial value problem

$$y'' - y = xe^x \quad \text{with } y(0) = 1 \text{ and } y'(0) = -\frac{1}{4}.$$