# THE DUFFING OSCILLAT OR

Amit Eshed MATH 2410Q-001



# THE EQUATION

Second order non-linear Differential Equation

Models springs or any form of oscillators that don't follow Hooke's Law

Used to model damped, driven, and non-linear oscillators and springs

Popular uses for Chaos Research

## LINEAR VERSUS NON-LINEAR SPRINGS?

Linear

Linear Restoring Force

Hooke's Law

#### **Non-linear/Progressive**

Non-linear Restoring Force







### BREAKDOWN OF EQUATION

By Terms and Variables

# THE EQUATION (BY TERMS)

If is the displacement of a spring in respect to time

Acceleration

Velocity

. . .

Displacement

Dampened Harmonic Oscillator

Non-linear Restoring Force

**Driving Force** 

# THE EQUATION (BY VARIABLES)

Where the coefficients and variables are:

Amount of dampening

Linear Stiffness (k in Hooke's Law)

Non-linear stiffness

Amplitude of driving force

Frequency of driving force



### APPLICATIONS AND SOLUTIONS

Models and Graphs



# VISUALIZING THE SOLUTION

Using graphs

- Displacement as a function of time
- Velocity as a function of displacement

#### **EXAMPLE SOLUTION GRAPHS** $x'' + \delta x' + \alpha x + \beta x^3 = \gamma \cos(\omega t)$

 $x'' + x = \cos(t)$ 



If:  $\delta = \beta = 0$  $\alpha = \gamma = \omega = 1$ 

Solution: Simple, Harmonic No non-linear, no dampening

#### **EXAMPLE SOLUTION GRAPHS** $x'' + \delta x' + \alpha x + \beta x^3 = \gamma \cos(\omega t)$

$$x'' + x' + x = \cos(t)$$



If:  $\beta = 0$  $\delta = \alpha = \gamma = \omega = 1$ 

Solution: Simple, Harmonic No non-linear

#### **EXAMPLE SOLUTION CDVDHC** $\gamma = 1$ x'х х 0 x'х t х M $\gamma = 100$ x' х

t

х

If:  $\gamma = \text{variable}$   $\delta = \alpha = \beta = \omega = 1$ x'(0) = 1



### HISTORY

Applications and the Inventor

# **APPLICATIONS**

Springs

Oscillators

Chaotic Systems





GEORG DUFFING

1878-1883 Polytechnic Karlsruhe in Mechanical Engineering

1910 Patents in US

1913 Attended Max Plank's Lectures in Berlin

1918 Published Duffing Equation

1929 Worked for Ossag (Shell)

1944 Leaves Berlin due to Airraids and passes away in the countryside



## REFERENCES

- Rimrott F. (1994). Georg Duffing (1861-1944). Technical Mechanics, L4, 77-82. Retrieved November 5, 2018, from http://www.uni-magdeburg.de/ifme/zeitschrift\_tm/1994\_Heft1/Rimrott\_2.pdf
- Southern Atlantic Springs. (n.d.). Compression Spring [Digital image]. Retrieved November 4, 2018, from https://www.southernatlanticspring.com/
- Wikipedia contributors. (2018, June 06). Georg Duffing. In Wikipedia, The Free Encyclopedia. Retrieved November 5, 2018, from https://de.wikipedia.org/wiki/Georg\_Duffing
- Wikipedia contributors. (2018, March 17). Duffing equation. In Wikipedia, The Free Encyclopedia. Retrieved 16:45, November 4, 2018, from <u>https://en.wikipedia.org/w/index.php?</u> <u>title=Duffing\_equation&oldid=830885674</u>

Zhao, F. (2014, April 22). Chaotic behavior of the Duffing Oscillator. pp. 1-11.