# The Bellman Equations

By Dawson Hettrick

## **Richard E. Bellman**

- Lived 1920-1984
- Born in New York
  - BA at Brooklyn college
  - MA at University of Wisconsin
  - Ph.D at Princeton
- American applied mathematician
- RAND
  - Introduced dynamic programming
- Bellman prize in Mathematical Biosciences



# Dynamic programming

**Dynamic Programming Algorithm** 

goal

1. Characterize the structure of an optimal solution

20

25

5

11

start

- 2. Recursively define the value of an optimal solution
- 3. Compute the value of an optimal solution in a bottom-up fashion
- Construct an optimal solution from computed information (not always necessary)

- Finding a solution to a problem by breaking the problem into multiple smaller problems recursively
- Can be used in math and coding
- Relationship between smaller subproblems and original problem is called the Bellman equation





- Merton's portfolio problem
  - Investors choose between income today and future income
- Economic growth
- Taxation
- AI learning
- Reinforcement learning



#### observation

### Important values



s=state

## Value functions

State value function:  $\bigvee$ 

$$/^{TT}(s) = \mathbb{E}_{\pi} \left[ \mathbb{R}_{+} \right] s_{+} = s \right]$$

Action value function:  $Q^{\dagger}(s, a) = \mathbb{E}_{\pi} [R_{+}|s_{+}=s, a_{+}=a]$ 

# Bellman equation derivation

https://en.wikipedia.org/wiki/Richard\_E.\_Bellman

https://joshgreaves.com/reinforcement-learning/understanding-rl-the-bellmanequations/

https://en.wikipedia.org/wiki/Bellman\_equation