Bernoulli’s Equations

By Jeff Remy

What is Bernoulli's Equations?

$$y' + p(t)y = q(t)y^n$$

How to solve a generic Bernoulli's DE?

$$y' + p(t)y = q(t)y^n$$

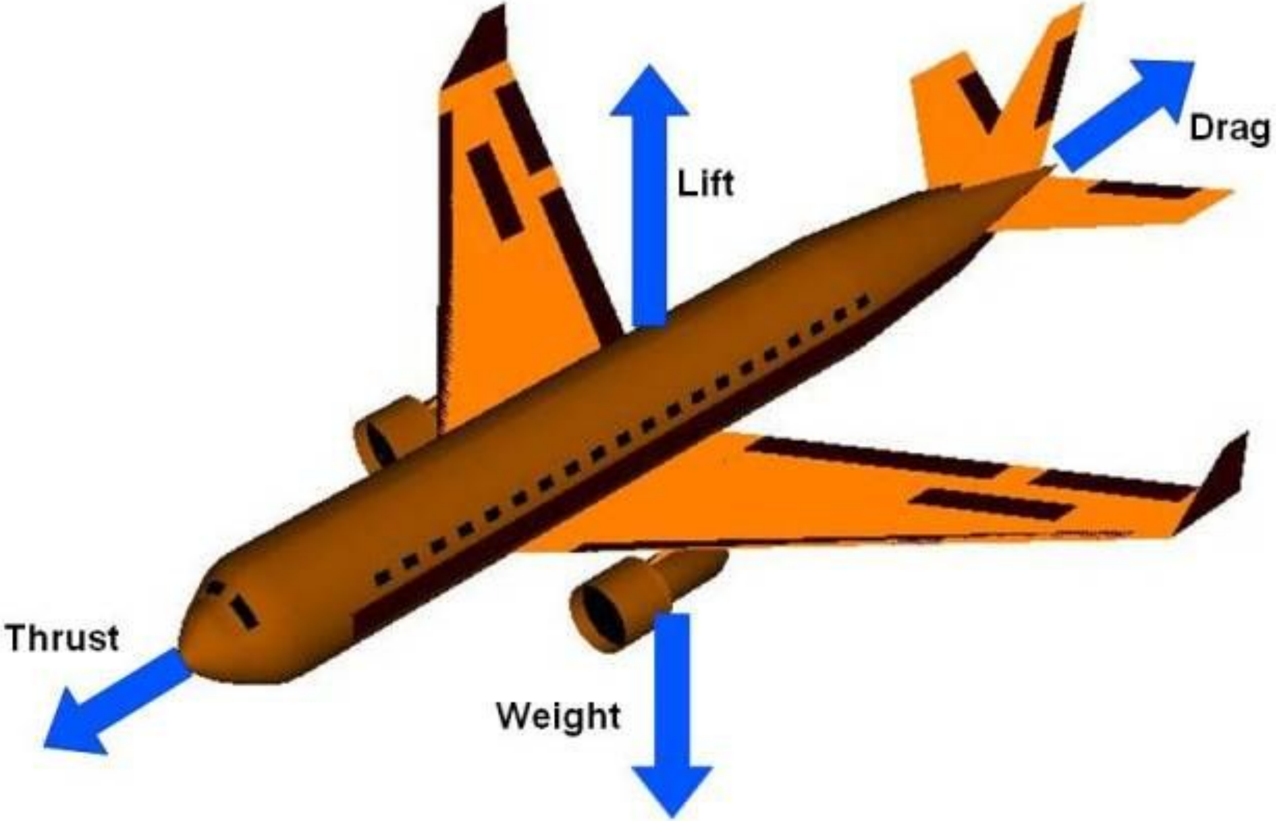
1. Use the substitution $u = y^{(1-n)}$
2. Find u'
3. Substitute corresponding parts
4. Equation should now be in linear form

Applications of Bernoulli’s Equations

1. Fluid flow
2. Logistic Growth Equation

Fluid Flow

$$p + \frac{1}{2}\rho V^2 + \rho g h = constant$$


Lift
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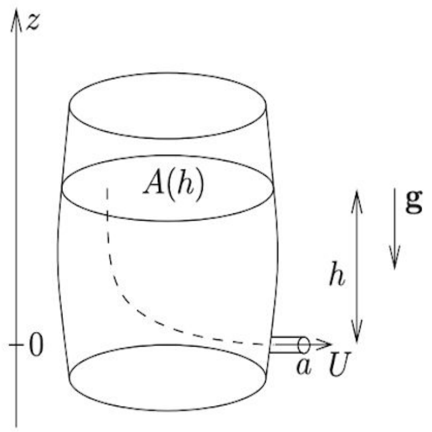

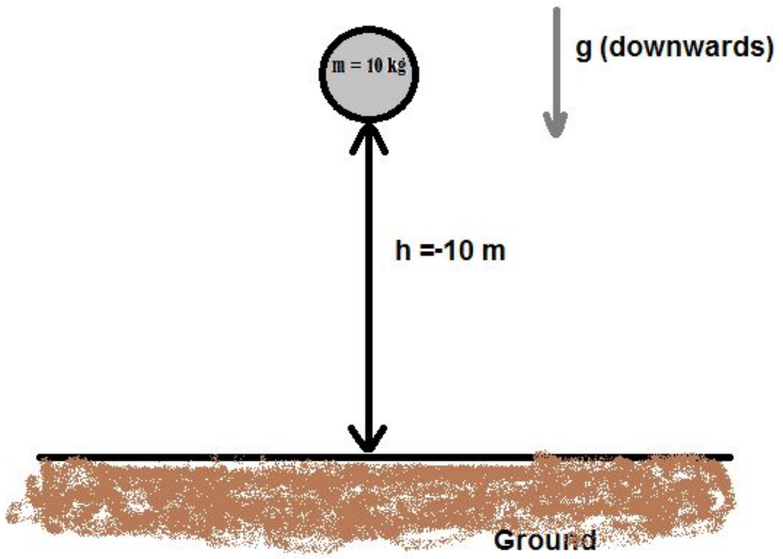

$$p + \frac{1}{2}\rho V^2 + \rho g h = constant$$

Solve With the Class

Unsteady potential flow

: is used in the theory of ocean surface waves and acoustics.

$$\frac{\partial \varphi}{\partial t} + \tfrac{1}{2} v^2 + \frac{p}{\rho} + gz = f(t),$$

which is a Bernoulli equation valid also for unsteady—or time dependent—flows. Here $\partial\varphi/\partial t$ denotes the partial derivative of the velocity potential φ with respect to time t, and $v = |\nabla \varphi|$ is the flow speed

Logistic Growth Models

- The solution of the general logistic differential equation

$$\frac{dP}{dt} = kP(M - P)$$

is

$$P = \frac{M}{1 + Ae^{-(Mk)t}}$$

where A is a constant determined by an appropriate initial condition. The **carrying capacity** M and the **growth constant** k are positive constants.

Citations

Merton, Robert C. “Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case.” *The Review of Economics and Statistics*, vol. 51, no. 3, 1969, pp. 247–257. *JSTOR*, JSTOR, www.jstor.org/stable/1926560.

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