Euler’s Equation of Inviscid Motion

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Leonhard Euler

- Swiss mathematician, physicist, astronomer, logician, and engineer
- Made discoveries in mathematics, mechanics, fluid dynamics, optics, astronomy, and music theory
- Student with Daniel Bernoulli and studied from Johann Bernoulli
- Studied various fluid dynamic problems in the mid- 1700’s

The Equation

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = - \frac{\nabla P}{\rho}$$

- **u** is the fluid velocity
- **P** is the pressure
- **ρ** is the fluid density
- **t** is the time

What it Represents

- Describes fluid flow in the absence of viscosity
- Incompressible fluid
- Condition to the equation:
  - $\nabla \cdot u = 0$
- Compressibility
  - $B = (1/\rho)(d\rho/dp)$
  - If compressibility is small, flow is incompressible

Other Equations Where It Can Be Derived From

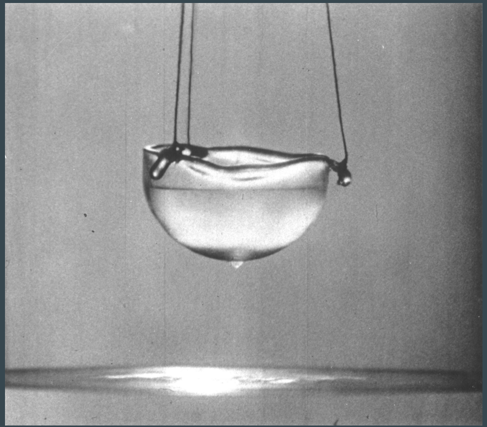
$$\begin{cases} \dfrac{D\mathbf{u}}{Dt} = -\nabla w + \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

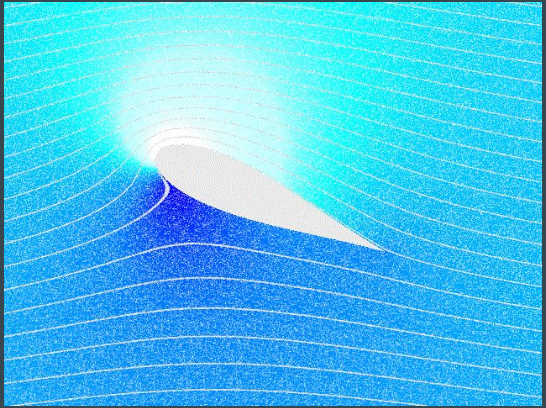
$$\begin{cases} \dfrac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla w + \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

$$\nabla w \equiv \nabla \left(\frac{p}{\rho_0} \right) = \frac{1}{\rho_0} \nabla p$$

Applications

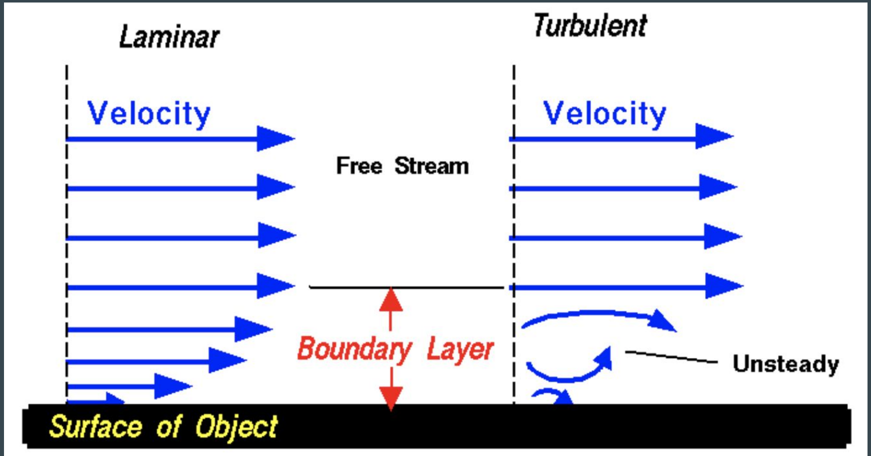
- Superfluids
  - State of matter that exhibit frictionless flow (zero viscosity)
  - Helium is a superfluid once cooled to 2.2 K
  - Spectrometers
- Fluid Dynamics
  - Instances in which viscosity is insignificant
    - Flow of air around airplane wing
    - Ocean currents
    - Flow around bridge supports in a river





Limitations

- Since most fluids are not inviscid fluids, Euler equation is an approximation of real fluid problems
- Example in which it would not work
  - Growth of a boundary layer on a flat plate (velocity changes from 0) because it occurs on the boundary of a fluid



Relation to the Navier-Stokes Equation

$$\rho \frac{Dv}{Dt} = -\nabla p + \mu \nabla^2 v + \rho g$$

- Developed by Claude-Louis Navier and published by George Gabriel Stokes
- Describes the motion of fluids where viscosity is negligible (not 0)
- Euler equation is a simplification
  - Reduces to Euler's Equation when $\mu = 0$

Sources

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