Chandrasekhar’s White Dwarf Equation

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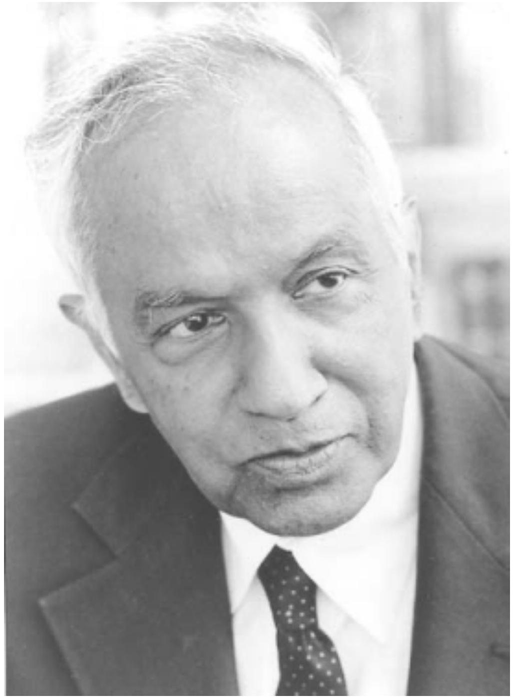
What we’ll be covering...

- Introduction to Chandrasekhar
- The Chandrasekhar White Dwarf Equation
- The elements and variables of the equation
- What the equation represents
- The derivation
- The applications of the equation and The Chandrasekhar Limit

Subrahmanyan Chandrasekhar

- Indian American Astrophysicist
- Ph.D. in physics from Cambridge
- Won the Nobel Prize in 1983 for "...theoretical studies of the physical processes of importance to the structure and evolution of the stars".
- Produced many of the best theoretical models of the later evolutionary stages of massive stars and black holes



$$\frac{1}{\eta^2}\frac{d}{d\eta}\left(\eta^2\frac{d\varphi}{d\eta}\right)+(\varphi^2-C)^{3/2}=0$$

with initial conditions

$$\varphi(0)=1,\quad \varphi'(0)=0$$

- Φ (phi) is the density of the white dwarf
- η (eta) is the dimensionless radius
- *C* is a constant related to the density

What it represents

- Shows relationship and behavior of the density of the white dwarf as a function of radius
- Subject to initial value conditions of $\phi(0) = 1$ and $\phi'(0) = 0$
- Governs the structure of degenerate matter in gravitational equilibrium

Derivation

Hydrostatic equilibrium equations

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho,$$

$$\frac{dM_r}{dr} = 4\pi r^2 \rho.$$

Poisson equation for gravity

$$\nabla \cdot \mathbf{g} = -4\pi G\rho \, .$$

Equation for a spherical polytrope (second order Poisson equation)

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho$$

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho$$

Pressure and density of a white dwarf

$$P = Af(x), \quad \rho = Bx^3$$

where

$$A = 6.01 \times 10^{22},\ B = 9.82 \times 10^5 \mu_e,$$
$$f(x) = x(2x^2 - 3)(x^2 + 1)^{1/2} + 3\sinh^{-1} x$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dy}{dr}\right) = -\frac{\pi G B^2}{2A}(y^2-1)^{3/2}$$

Use the dimensionless scale

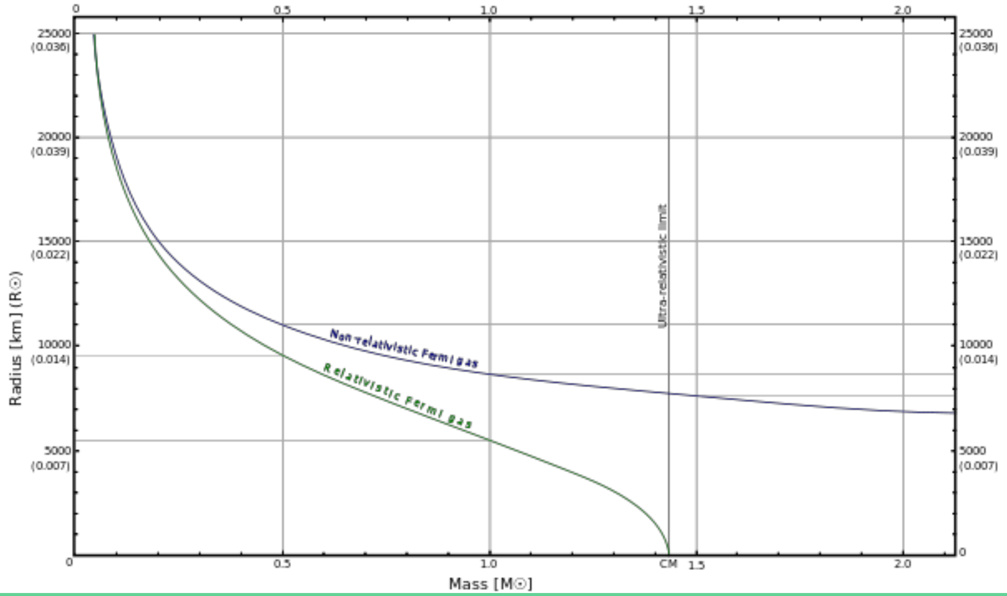
$$r = \left(\frac{2A}{\pi G B^2}\right)^{1/2}\frac{\eta}{y_o}, \quad y = y_o\varphi$$

$$\Downarrow$$

$$\frac{1}{\eta^2}\frac{d}{d\eta}\left(\eta^2\frac{d\varphi}{d\eta}\right) + (\varphi^2 - C)^{3/2} = 0$$

Applications

Given the radius we can determine the density of a white dwarf star. This information can in turn help determine the maximum mass of the star and other properties of degenerate matter. This is seen in Chandrasekhar's Limit Equation.



$$M_{\text{limit}} = \frac{\omega_3^0 \sqrt{3\pi}}{2} \left(\frac{\hbar c}{G}\right)^{\frac{3}{2}} \frac{1}{(\mu_e m_H)^2}$$

Works Cited

https://en.wikipedia.org/wiki/Chandrasekhar%27s_white_dwarf_equation

https://en.wikipedia.org/wiki/Subrahmanyan_Chandrasekhar#Philosophy_of_systematization

http://www.fisica.edu.uy/~sbruzzone/FlexPaper_1.4.2_flash/prueba.pdf

http://www-astro.physics.ox.ac.uk/~podsi/problem_grad_diff.pdf

https://en.wikipedia.org/wiki/Hydrostatic_equilibrium

https://en.wikipedia.org/wiki/Poisson%27s_equation

https://www.astro.princeton.edu/~gk/A403/polytrop.pdf

S. Chandrasekhar; The Highly Collapsed Configurations of a Stellar Mass. (Second Paper.), *Monthly Notices of the Royal Astronomical Society*, Volume 95, Issue 3, 1 January 1935, Pages 207–225