Lane–Emden equation

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Learning Outcomes

- About Lane-Emden Equation
- The meaning of each symbol and variables on the equation and Application of Lane-Emden Equation
- Derivation the Lane-Emden Equation
- Solving Lane-Emden Equation
- Biography

- In astrophysics, the Lane–Emden equation is a dimensionless form of Poisson's equation for the gravitational potential of a Newtonian self-gravitating, spherically symmetric, polytrophic fluid. It is named after astrophysicists Jonathan Homer Lane and Robert Emden.

- The Lane–Emden equation was the first to describe the internal structure of a self-gravitating polytrophic body. When temperature effects are also combined, such as in the study of isothermal gas spheres, the astrophysical phenomenon is described by a Lane–Emden equation

- A second-order ordinary differential equation arising in the study of stellar interiors, also called the polytrophic differential equations.

- Where ξ (xi) is a dimensionless radius and θ *is related to the density*

by $\rho = \rho_c \theta^n$ for central density ρ_c

- The index n is the polytrophic index that appears in the polytrophic equation of state ,
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$$P = K\rho^{1+\frac{1}{n}}$$

- Where ***P*** *and* ***p*** *are the pressure and density , respectively , and* ***K*** *is a constant of proportionality .*

Derivation Lane–Emden equation

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho,$$

$$\frac{dM_r}{dr} = 4\pi r^2 \rho.$$

- We assume that a spherical star is in a hydrostatic equilibrium (The balance between Gravity in word and pressure out ward).

- These may be combined to a single, second order Poisson equation:

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho.$$

$$P = K\rho^{1+\frac{1}{n}}$$

$$\rho = \rho_c\theta^n, \quad P = P_c\theta^{n+1}, \quad r = \alpha\xi,$$

$$\alpha^2 = \frac{K(n+1)\rho_c^{\frac{1-n}{n}}}{4\pi G},$$

- We assume that there is a polytrophic relation between pressure and density:

- where K and n are real, positive constants, and n is called a polytrophic index. We introduce dimensionless variables:

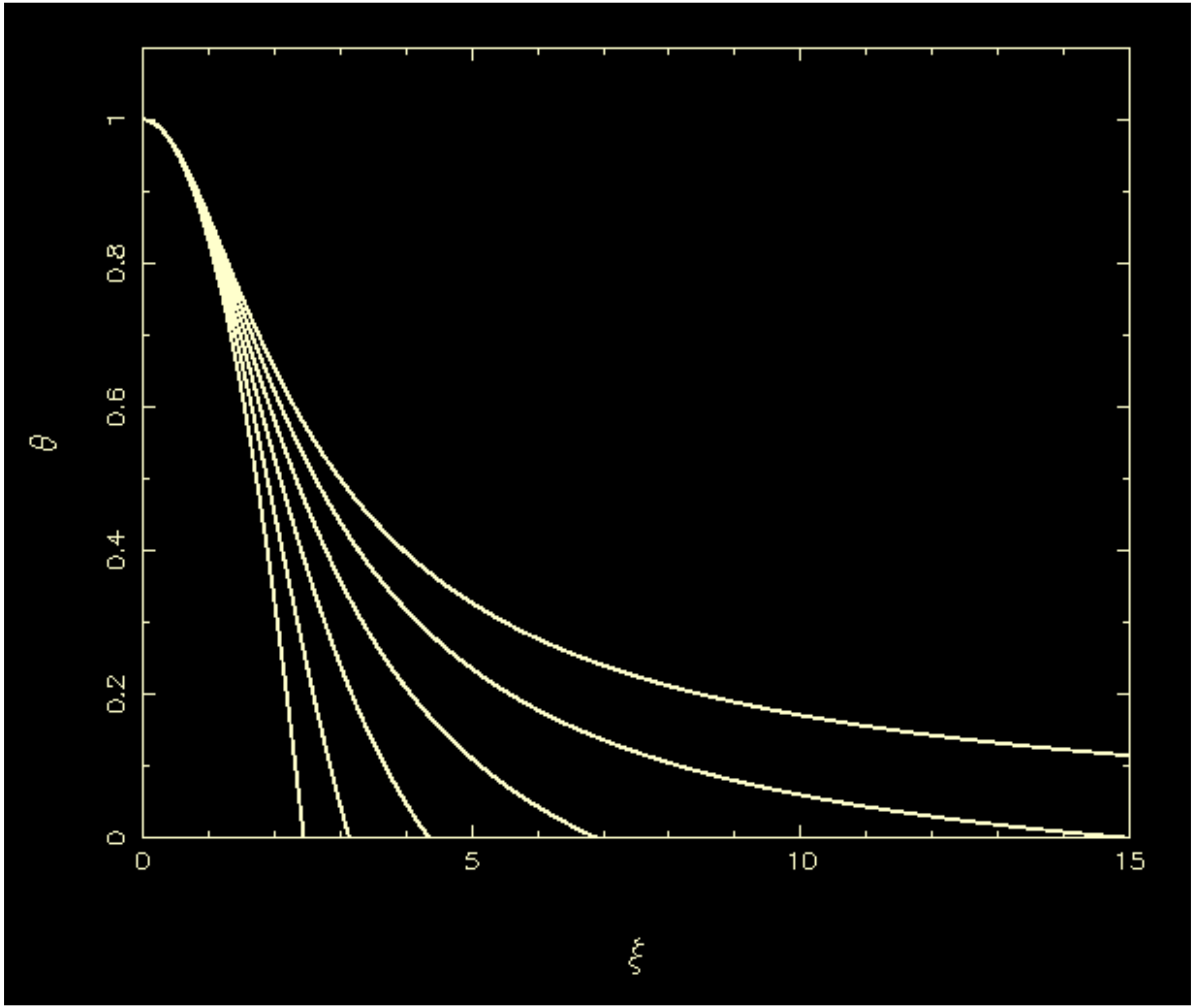
- And ξ is a new radius like variable .Combining equations .We obtain the Poisson equation in dimensionless variables .

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n.$$

Solving Lane -Emden equation

- It is possible to solve Lane Emden equation analytically .i.e. to drive a simple mathematical expression for the dimensionless density θ as a functionless radius ξ , for only three values of the polytrophic index , n . These are
- n = 0 , 1 , 5
- Solutions for all other values of n must be obtained numerically i.e. using computer to plot θ versus ξ .

Numerical Solution to the Lane _Emden equation for (Left-to-right) N=0,1,2,3,4,5



Power series solution of the Lane -Emden Equation

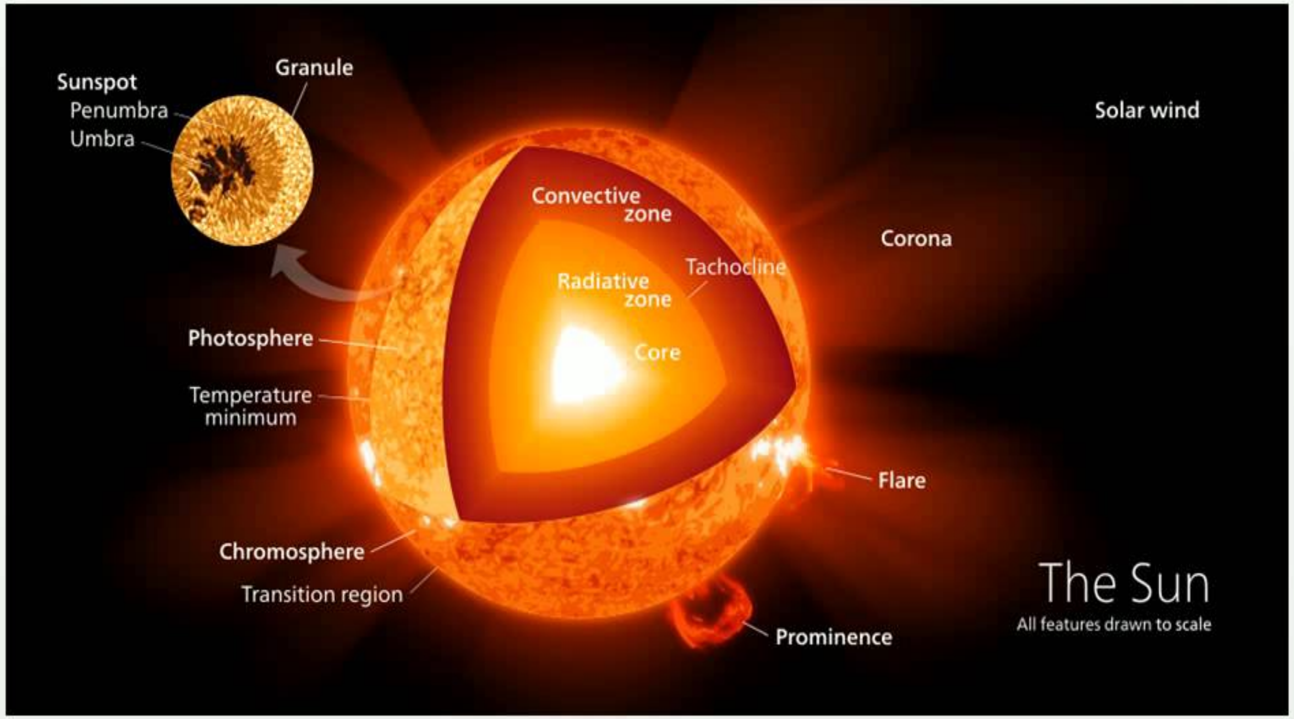
Biography

- Lane's parents were Mark and Henrietta (née Tenny) Lane and his education was at the Phillips Exeter Academy in Exeter, New Hampshire. He graduated from Yale University in 1846. He worked for the U.S. Patent Office, and became a principal examiner in 1851.
- Lane was particularly interested in astronomy, and was the first to perform a mathematical analysis of the Sun as a gaseous body.

Biography

- Emden was born in St. Gallen, Switzerland. In 1907 he became associate professor of physics and meteorology at the Technical University of Munich. The same year he published the classic work, *Gaskugeln: Anwendungen der mechanischen Wärmetheorie auf kosmologische und meteorologische Probleme* (Gas balls: Applications of the mechanical heat theory to cosmological and meteorological problems), which provided a mathematical model as a basis of stellar structure..

Stellar structure



https://en.wikipedia.org/wiki/File:Sun_poster.svg

Citation

- *Vik Dhillon: phy213 - the Structure of Main-Sequence Stars - Solving the Lane-Emden Equation*, www.vikdhillon.staff.shef.ac.uk/teaching/phy213/phy213_le.html.
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- "Lane–Emden Equation." *Wikipedia*, Wikimedia Foundation, 7 Oct. 2018, en.wikipedia.org/wiki/Lane%E2%80%93Emden_equation.
- Mirza, and Babur M. "Approximate Analytical Solutions of the Lane-Emden Equation for a Self-Gravitating Isothermal Gas Sphere." *OUP Academic*, Oxford University Press, 15 May 2009, academic.oup.com/mnras/article/395/4/2288/972369.