Fall 2018 - Math 3150
Exam 1 - September 27
Time Limit: 75 Minutes

Name (Print): ______________________

This exam contains 6 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	13	
2	10	
3	10	
4	12	
5	10	
6	10	
7	10	
Total:	75	

1. For each of the following statements, say whether it is true or false. If the statement is false, give a counterexample.
   (a) (3 points) The sum of two irrational numbers is irrational.

   (b) (4 points) $\sqrt[3]{216}$ is an irrational number.

   (c) (3 points) Convergent sequences are bounded.

   (d) (3 points) If (s_n) and (t_n) are convergent sequences of real numbers with $\lim s_n = s$ and $\lim t_n = t$. If $s_n < t_n$ for every $n \in \mathbb{N}$ then $s < t$.

2. (10 points) Let (s_n) be a sequence of real numbers for $n \in \mathbb{N}$.

   Show that $\lim s_n = 0$ if and only if $\lim |s_n| = 0$.

3. (10 points) Prove that $\sqrt{1+\sqrt{2}}$ is irrational.

4. Let $S = \{x \text{ irrational};\ 0 \leq x \leq 2\}$.

   (a) (4 points) What are $\sup(S)$ and $\inf(S)$?

   (b) (4 points) Does the set S have a maximum element and/or a minimum element?

   (c) (4 points) Write a decreasing sequence (s_n) such that $s_n \in S$ for each $n \in \mathbb{N}$ for which (s_n) converges to $\inf(S)$. Here decreasing means that $s_n \geq s_{n+1}$ for every $n \in \mathbb{N}$.

5. (10 points) Let (t_n) and (s_n) be two sequences of real numbers for which t_n converges to t and s_n converges to s. Prove that $\lim(t_n + s_n) = s + t$.

6. (10 points) Using ϵ-definition, prove that

$$\lim \frac{2n-1}{n^2+1} = 0.$$

7. (10 points) Using the definition 9.8, (given $M > 0$, there is a number N such that $n > N$ implies $s_n > M$) prove that

$$\lim \frac{n^3+3150}{n+2018} = \infty.$$