UNIVERSITY OF CONNECTICUT

Fall 2018 - Math 3150
Exam 2 - October 30
Time Limit: 75 Minutes

Name (Print): ____________________

This exam contains 8 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	12	
2	12	
3	19	
4	10	
5	12	
6	10	
7	0	
Total:	75	

1. For each of the following statements, say whether it is true or false. If the statement is false, give a counterexample.

   (a) (4 points) For all sequences of real numbers (s_n) we have $\liminf s_n \leq \limsup s_n$.

   (b) (4 points) Every monotone sequence of real numbers is convergent.

   (c) (4 points) Every bounded sequence of real numbers has at least one convergent subsequence.

2. If possible, give an example of each of the following. Write "not possible" when appropriate.

   (a) (4 points) A sequence (s_n) with $\limsup s_n = \infty$ and $\liminf s_n = 0$.

   (b) (4 points) A bounded sequence which is not convergent.

   (c) (4 points) Give an example of a bounded sequence of real numbers with exactly two subsequential limits.

3. Let $s_1 = 1$ and $s_{n+1} = \frac{1}{3}(s_n + 1)$ for $n \geq 1$.

   (a) (3 points) Find s_2, s_3, and s_4.

   (b) (4 points) Use induction to show $s_n > 1/2$ for all $n \in \mathbb{N}$.

   (c) (4 points) Show (s_n) is a decreasing sequence.

   (d) (4 points) Show $\lim s_n$ exists and find $\lim s_n$.

   (e) (4 points) Is (s_n) a Cauchy sequence?

4. (10 points) Let (s_n) be any sequence. There exists a monotonic subsequence whose limit is $\liminf s_n$.

5. Let $S = \{1/n : n \in \mathbb{N}\}$.

   (a) (4 points) Prove that S is not closed.

   (b) (4 points) Prove that S is not open.

   (c) (4 points) Prove that $S \cup \{0\}$ is compact.

6. Let f be a function on $[0,1]$ defined by

$$f(x) = \begin{cases} x & \text{when } x \in [0,1] \text{ rational,} \\ 0 & \text{when } x \in [0,1] \text{ irrational.} \end{cases}$$

(a) (5 points) Use $\epsilon - \delta$ definition to show that f is continuous at $x = 0$.

(b) (5 points) Use $\epsilon - \delta$ definition to show that f is discontinuous at all other rational points in $(0,1]$.

7. Let (s_n) be a bounded sequnece of real numbers.
   (a) (4 points (bonus)) Carefully state the definition of $\limsup s_n$ and $\liminf s_n$.

   (b) (4 points (bonus)) If $s_n = (-1)^n$, find $\limsup s_n$ and $\liminf s_n$.

   (c) (4 points (bonus)) If $t_n = \sin(n\pi/2)$ then find all subsequential limits of $(t_n s_n)$.