UCONN
UNIVERSITY OF CONNECTICUT

Fall 2018 - Math 3150
Exam 3 - December 6
Time Limit: 75 Minutes

Name (Print): ____________________

This exam contains 8 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	12	
2	12	
3	12	
4	15	
5	12	
6	12	
7	0	
Total:	75	

1. Define $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} x\sin(\frac{1}{x}) & \text{when } x \neq 0, \\ 0 & \text{when } x = 0. \end{cases}$$

   (a) (6 points) Show that f is continuous and uniformly continuous on $[-1, 1]$.

   (b) (6 points) Show that f is not differentiable at $x = 0$.

2. (12 points) Prove that every continuous function on $[a, b]$ is integrable on $[a, b]$.

3. Consider
$$f(x) = \int_0^{x^2} e^{\sqrt{t}} dt \quad \text{for } x \in [0, \infty).$$

(a) (4 points) Compute $f(0)$.

(b) (8 points) Show that f is differentiable on $(0, +\infty)$ and compute $f'(x)$.

4. Define $f : [0, 2] \to \mathbb{R}$ by

$$f(x) = \begin{cases} 3150 & \text{when } x \neq 1, \\ 0 & \text{when } x = 1. \end{cases}$$

(a) (4 points) Compute the lower Riemann sum $L(f)$.

(b) (4 points) Compute the upper Riemann sum $U(f)$.

(c) (7 points) Show that f is integrable on $[0, 2]$ and find $\int_0^2 f(x)dx$.

5. (12 points) Suppose that $f : \mathbb{R} \to \mathbb{R}$ satisfies

$$|f(x) - f(y)| \leq C|x - y|^2 \quad \text{for all } x, y \in \mathbb{R}.$$

for some $C > 0$. Show that f is constant. (Hint: Compute first derivative of f).

6. (12 points) Let $f : [0, +\infty) \to \mathbb{R}$ be a continuous and differentiable function satisfying

$$f(x) + xf'(x) \geq 0 \quad \text{for all } x > 0.$$

Show that $f(x) \geq 0$ for all $x \geq 0$. (Consider a function $g(x) = xf(x)$).

7. (a) (8 points (bonus)) Let f and g be a continuous functions on $[a, b]$ such that $\int_a^b f = \int_a^b g$. Prove that there exists $x \in [a, b]$ such that $f(x) = g(x)$.

   (b) (8 points (bonus)) Construct an example of integrable functions f and g on $[a, b]$ where $\int_a^b f = \int_a^b g$ but $f(x) \neq g(x)$ for all $x \in [a, b]$.