Fall 2018 - Math 3150
Practice Exam 1 - September 18
Time Limit: 75 Minutes

Name (Print): ____________________

This exam contains 7 pages (including this cover page) and 8 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	9	
2	10	
3	10	
4	9	
5	10	
6	7	
7	10	
8	10	
Total:	75	

1. For each of the following statements, say whether it is true or false. If the statement is false, give a counterexample.

   (a) (3 points) Convergent sequences are bounded.

   (b) (3 points) Every bounded sequence is convergent.

   (c) (3 points) The limit of a convergent sequence of negative numbers is negative.

2. (10 points) Using mathematical induction prove that $n^2 + 5n + 1$ is an odd integer for every positive integer n.

3. (10 points) Prove that $1 + \sqrt{1 + \sqrt{2}}$ is irrational.

4. Let $S = \{x \text{ irrational};\ 1 \leq x \leq 3\}$.

   (a) (3 points) What are $\sup(S)$ and $\inf(S)$?

   (b) (3 points) Does the set S have a maximum element and/or a minimum element?

   (c) (3 points) Write an increasing sequence $s_n \in S$, i.e. $s_n \leq s_{n+1}$ for every $n \in \mathbb{N}$ for which s_n converges to $\sup(S)$. (You found $\sup(S)$ in part (a)].

5. (10 points) Consider the sequence (s_n) defined for $n \in \mathbb{N}$ by

$$s_n = \begin{cases} 1 & \text{when } n \text{ is odd,} \\ 2 & \text{when } n \text{ is even.} \end{cases}$$

Show that (s_n) does not converge.

6. (7 points) Suppose that $\lim a_n = a$ and $\lim b_n = b$ for some real numbers a and b.

$$\text{Find } \lim \frac{a_n^{3150} + b_n^{2018}}{a_n^2 + b_n^2 + 1}.$$

Justify all steps.

7. (10 points) Using ϵ-definition, show that

$$\lim \frac{n-5}{n^2+7} = 0.$$

8. (10 points) Using the definition 9.8, (given $M > 0$, there is a number N such that $n > N$ implies $s_n > M$) show that

$$\lim \frac{n^2 + 3}{n + 1} = \infty.$$