UCONN
UNIVERSITY OF CONNECTICUT

Fall 2018 - Math 3150
Practice Exam 2 - October 30
Time Limit: 75 Minutes

Name (Print): ______________________

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	16	
2	13	
3	12	
4	12	
5	10	
6	12	
Total:	75	

1. Let $s_1 = 1$ and $s_{n+1} = [1 - \frac{1}{(n+1)^2}]s_n$ for $n \geq 1$.

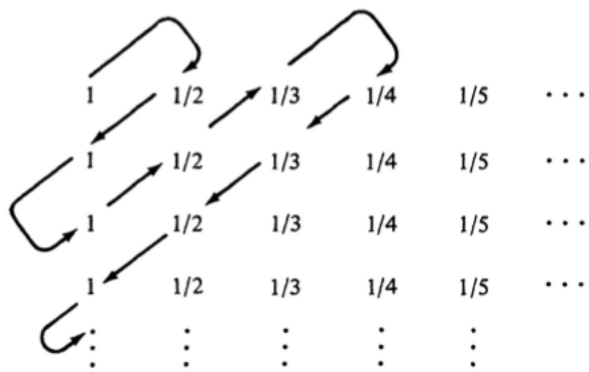
   (a) (4 points) Find s_2, s_3, and s_4.

   (b) (8 points) Use induction to show $s_n = \frac{n+1}{2n}$ for all $n \in \mathbb{N}$.

   (c) (4 points) Find $\lim s_n$.

2. (13 points) Let (a_n) and (b_n) be two convergent sequences of real numbers both converging to the same real number $r \in \mathbb{R}$. Let (c_n) be a sequence defined as $c_{2n-1} = a_n$ and $c_{2n} = b_n$ for all $n \in \mathbb{N}$, i.e., $(c_n) = (a_1, b_1, a_2, b_2, \ldots,)$. Then show that $(c_n) \to r$.

3. Let (s_n) be the sequence given in the figure.



(a) (6 points) Find the set S of the subsequential limits of (s_n).

(b) (6 points) Determine $\limsup s_n$ and $\liminf s_n$.

4. (a) (6 points) Find a decreasing sequence of (non empty) closed sets $F_1 \supset F_2 \supset \cdots \supset F_n \supset F_{n+1} \supset \dots$ such that

$$\cap_{n=1}^{\infty} F_n = \emptyset.$$

(b) (6 points) Find a decreasing sequence of (non empty) open bounded intervals $I_1 \supset I_2 \supset \cdots \supset I_n \supset I_{n+1} \supset \dots$ such that

$$\cap_{n=1}^{\infty} I_n = \emptyset.$$

5. (10 points) Let (s_n) be any sequence. Show that there exists a monotonic subsequence whose limit is $\limsup s_n$.

6. (12 points) The Dirichlet function $f : \mathbb{R} \mapsto \mathbb{R}$ defined by

$$f(x) = \begin{cases} 1 & \text{when } x \in \mathbb{Q}, \\ 0 & \text{when } x \notin \mathbb{Q}. \end{cases}$$

Show that f is discontinuous at every $x \in \mathbb{R}$.