



Fall 2018 - Math 3150
Practice Exam 2 - October 30
Time Limit: 75 Minutes

Name (Print): _____

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- If you use a “fundamental theorem” you must indicate this and explain why the theorem may be applied.
- Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- Mysterious or unsupported answers will not receive full credit. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	12	
2	12	
3	12	
4	15	
5	12	
6	12	
Total:	75	

Do not write in the table to the right.

1. Define $f : \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x^2}\right) & \text{when } x \neq 0, \\ 0 & \text{when } x = 0. \end{cases}$$

- (a) (6 points) Find $f'(x)$ for $x \in \mathbb{R} \setminus \{0\}$.
- (b) (6 points) Show that f is differentiable at $x = 0$ and find $f'(0)$.
- (c) (5 points (bonus)) Show that f' is not continuous at $x = 0$.

2. (12 points) Prove that every monotone function on $[a, b]$ is integrable on $[a, b]$.

3. Let $f : [-\pi, \pi] \rightarrow \mathbb{R}$ be the function defined by

$$f(x) = \int_0^{x^2} e^{\sin(t)} dt.$$

- (a) (4 points) Compute $f(0)$.
- (b) (8 points) Show that f is differentiable and compute $f'(x)$.

4. Define $f : [0, 1] \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} 2 & \text{when } x \neq \frac{1}{2}, \\ 0 & \text{when } x = \frac{1}{2}. \end{cases}$$

- (a) (4 points) Compute the lower Riemann sum $L(f)$.
- (b) (4 points) Compute the upper Riemann sum $U(f)$.
- (c) (7 points) Show that f is integrable on $[0, 1]$ and find $\int_0^1 f(x)dx$.

5. (a) (6 points) Show that for all $x, y \in \mathbb{R}$

$$|\sin(x) - \sin(y)| \leq |x - y|.$$

- (b) (6 points) Is the function $f(x) = \sin(x)$ uniformly continuous on \mathbb{R} ?

6. (12 points) Prove that $1 + x < e^x$ for all $x > 0$.