MATH 3410

Notes For August 30 and September 1

Lecture 1: The Differential Equations

Solve ~~[illegible]~~ the following quadratic equation

- $x-1=0$

This has no soln

- $x^2-3x+2=0$

$(x-1)(x-2)$

x^2-3x+2

$x=1$ and $x=2$ are solutions

- $x^2-2x+2=(x-1)^2+1$
- $x^2+1=0$

all x are solns

has no soln.

Lesson 3A: The Differential Equations

let $f(x)=\log x = y(x)$ and

$$f'(x)=\frac{1}{x}=y'=\frac{dy}{dx}=D_x f(x)$$

$$f''(x)=-\frac{1}{x^2}, \quad f'''(x)=\frac{2}{x^3} \quad \text{etc.}$$

$$y'=\frac{1}{x}, \quad y''=-\frac{1}{x^2}$$ → this is a differential equation

$(y')^2=\left(\frac{1}{x}\right)^2$ therefore $y''=-(y')^2$ is also differential eqn

Later in the semester our functions will have two or more variables $f(x,y) = \cos(x^2+y^2)$.

then we will talk about partial differential equations;

$$\frac{\partial f}{\partial x} = f_x = -2x\sin(x^2+y^2)$$

$$\frac{\partial f}{\partial y} = f_y = -2y\sin(x^2+y^2).$$

$$\frac{\partial^2 f}{\partial x\,\partial y} = -2x\cdot 2y\cos(x^2+y^2).$$

etc.

Defn 1: Let $f = f(x)$ be defined on an interval $I = a < x < b$. By an ordinary differential equation we mean an equation involving x, the function $f(x)$ and its derivatives.

Examples:

1. $y' + p(x)y = q(x)$ where p, q are cont. functions.
2. $xy' - 4y + 2x^2 + 4 = 0$
3. $y'' \cdot y' + y^2 + 2x = 0$
4. $(y^{(4)})^{10} + y = 0$

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The order of the DE is the highest order of the derivative involved in the equation.

1 → First order DE.
2 → First order DE
3 → Second order DE
4 → Fourth order DE.

Solution of ~~the~~ DEs.

Remember that when we have an algebraic equations $x^2-2x+1=0$, we can show that $x=1$ is a solution, that means when you substitute $x=3$ into this equation then the equality holds.

$$1^2-2\cdot 1+1=0$$: the equality holds.

In a similar fashion we will say that $f(x)$ is a solution of a differential equation it should satisfy it;

Example: $f(x)=x^2$ is a solution to the DE $x^2y''-2xy'+2y=0$

Since $f'(x) = 2x$ $f''(x) = 2$

Use these to substitute into the differential eqn

$$x^2 \cdot 2 - 2x \cdot 2x + 2 \cdot x^2 = 2x^2 - 4x^2 + 2x^2 = 0.$$

Hence $f(x) = x^2$ is a solution to the above DE.

Defn: "solve a DE" or "find a solution" to a DE" we mean to find a function $f(x)$ or $y(x)$ which solves the DE.

Example: Verify that the function defined by

$$y = x^2 \quad -\infty < x < \infty$$

is a solution to

$$(y'')^3 + (y')^2 - y - 3x^2 - 8 = 0 \quad (*)$$

Soln: Now since $y = x^2$ therefore $y' = 2x$ $y'' = 2$

Substitute this into above DE;

$$(2)^3 + (2x)^2 - x^2 - 3x^2 - 8 = 8 + 4x^2 - x^2 - 3x^2 - 8$$

$$= 0.$$

Hence $y = x^2$ is a solution to $y = x^2$ and $(*)$ is

Now since the function $y = x^2$ $y = x^2$ is solution well-defined for $x \in (-\infty, \infty)$ for $-\infty < x < \infty$

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Defn: Let $f(x)$ be defined on $a<x<b$. We say that $f(x)$ is an explicit solution or simply a solution of DE

$$F(x, y, y', \dots, y^{(n)}) = 0$$

if $F(x, f(x), f'(x), \dots, f^{(n)}(x)) = 0$, for any $x \in I$.

Example: Show that

$$y = \frac{1}{x^2-x-6}$$

is an explicit soln to $y' = (1-2x)y^2$. Find the longest interval that the solution is defined containing 0.

Soln: Since $y = \frac{1}{x^2-x-6}$ ~~found~~

$$F(x, y, y') = y' - (1-2x)y^2 = 0$$

and $y' = -\frac{1}{x^2-x-6} \cdot 2x-1 = (1-2x) \cdot y$

Equivalently $y' - y(1-2x) = 0$

Hence $F(x, y, y') = 0$. Therefore y is solution DE.

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