differential equation. Now differentiate the solu

$$y' = -c \sin x + 1$$

Now we need to get rid of c.

Solve for c in the solution

as $y = c\cos x + x$ then

$$\frac{y-x}{\cos x} = c \quad \text{use this in } y'$$

$$y' = -\frac{(y-x)}{\cos x} \cdot \sin x + 1 = -(y-x)\tan x + 1$$

$$= (x-y)\tan x + 1$$

therefore the DE we are looking for is

$$y' = (x-y)\tan x + 1$$

here we ~~do~~ $\tan x$ is undefined when

$$x = \mp\frac{\pi}{2}, \frac{3\pi}{2}, \dots$$

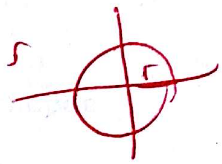
therefore $y' = (x-y)\tan x + 1 \quad . \; x \neq \mp\frac{\pi}{2}, \mp\frac{3\pi}{2}, \dots$

Example: Find a differential equation whose 1-parameter family of solutions represents a family of circles with centers at the origin.

Soln: The family of circles centered at zero can be written as

$$x^2+y^2=r^2, \quad r>0$$

To find the DE we get rid of r



$$2x+2y\cdot y'=0 \quad \text{or}$$

$$y'\cdot y=-x.$$

Lesson 4 c. General solution. Particular Soln. Initial Conditions.

A solution is called general solution of a DE if it's an n-parameter family of solutions of an nth order DE.

Examples: ① $y=ce^{x}$ is a general solution of the DE $y'-y=0$

② $y=c_1e^{-2x}+c_2e^{-x}+2e^{x}$ is a general solution to the DE $y''+3y'+2y-12e^{x}=0$

<u>Definition</u>: The n-conditions which enable us to determine the values of the arbitrary constants $c_1 \ldots c_n$ in an n-parameter of family ~~of solutions~~ if given in terms of ~~the~~ one value of the independent variable are called initial conditions.

<u>Example</u>: Find a 1-parameter family of solutions of the DE $dy = y\,dx$ and the particular solution for which $y(3)=1$

<u>Soln</u>: Since $y' = y$ or $\frac{y'}{y} = 1$

$$\log y = x + c$$

$$\text{or} \quad y = e^{x+c} = e^x \cdot \underbrace{e^c}_{c}$$

So 1-parameter family of soln is $y = ce^x$

In order to find the particular solution which satisfies $y(3)=1$

substitute $x = 3$, $y(3) = ce^3 = 1$ then solve for $c = e^{-3}$. Hence $y(x) = e^{-3} \cdot e^x = e^{x-3}$ is the particular soln.

Example: Sketch the direction field for the DE. sketch the integral curves for the this DE

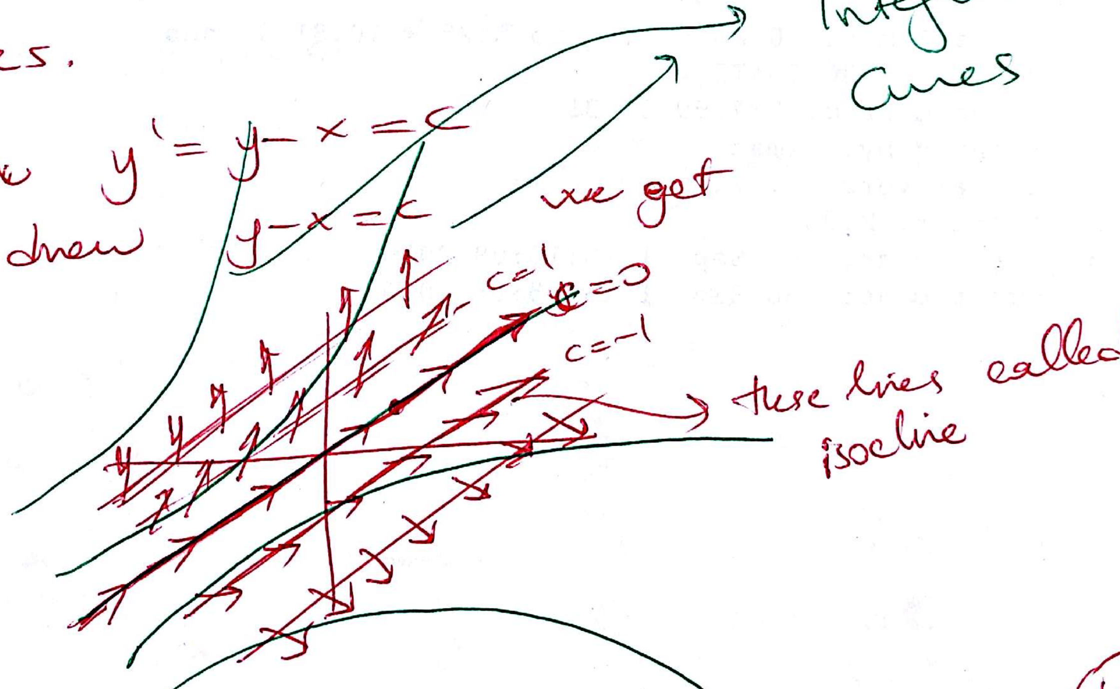
$$y' = y - x.$$

Solution: To sketch the direction field for this kind of DE we first identify places where the derivative will be constant. To do this we set the derivative in the DE equal to a constant, say c. This gives us a family of equations called isoclines.

Now $y' = y - x = c$

$y - x = c$

If you draw we get



Scanned by CamScanner

Special Types of First order ODE.

lesson 6C: Differential equations with Separable variables:

DEs which can be in the following form

$$Q(x,y)\frac{dy}{dx} + P(x,y) = 0$$

OR equivalently we can write

$$Q(x,y)\,dy + P(x,y)\,dx = 0$$

is called differential equation ~~will be~~ if

$$Q(x,y) = A(x).B(y)$$
$$P(x,y) = C(x).D(y)$$

Example: Find a 1-parameter family of solutions of the DE

$$x\sqrt{1-y}\,dx - \sqrt{1-x^2}\,dy = 0.$$

also a particular solution that cannot be obtainable from the family

Solution: The domain of the DE is when $y \geq 1$ & $1-x^2 \geq 0$ i.e. $-1 \leq x \leq 1$

The DE can be rewritten as when $x \neq \pm 1$, $y \neq 1$

$$\frac{x}{\sqrt{1-x^2}}\,dx - \frac{dy}{\sqrt{1-y}} = 0$$

$$-\sqrt{1-x^2} + \sqrt{1-y} = c \qquad -1 < x < 1, \quad y > 1$$

is the 1-parameter family of sols of the DE.

Moreover $y = 1$ is a particular solution

See http://www.math.uconn.edu/~akman/math3410f17/index.html
For your first assignment.