Fall 2016 - Math 3410
Exam 2 - November 4
Time Limit: 50 Minutes

Name (Print): ____________________

This exam contains 9 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	12	
3	12	
4	12	
5	24	
Total:	80	

[1]

[1] Exam template credit: `http://www-math.mit.edu/~psh`

1. Consider the following differential equation

$$y'' + k^2 y = 0 \quad \text{where } k \text{ is a constant.}$$

   (a) (3 points) Classify all points as ordinary, regular singular, or irregular singular.

   (b) (4 points) You are going to find a power series solution to above differential equation around $x_0 = 0$. As a first step, using power series method, find the recurrence relation. By checking some of the terms, try to find a pattern.

(c) (5 points) Using part (b) find the power series solution to the above differential equation. (Hint: combine even and odd terms).

(d) (4 points) Find the interval of convergence of the power series you found in (c).

(e) (4 points) Find the function representations of the power series you found in (c).

2. (12 points) Consider the *Van der Pol*s equation

$$y'' + \mu(y^2 - 1)y' + y = 0$$

arose as an idealized description of a spontaneously oscillating circuit. Use the **second method** to find first four terms of the power series solution $y(x) = \sum_{n=0}^{\infty} a_n x^n$ around $x_0 = 0$ with $\mu = 1$ with the initial conditions $y(0) = 0$ and $y'(0) = 1$.

$a_0 = ...$ $a_1 = ...$ $a_2 = ...$ $a_3 = ...$

Using your previous work, write the first four terms of $y(x) =$ .... + ... + ... + ... +

3. (12 points) Classify all the points as ordinary point, regular singular point, or irregular singular point for the following differential equation

$$(x-1)^3y'' - (x-1)y' + 4(x-1)y = 0.$$

4. Consider the following differential equation

$$2xy'' + y' + xy = 0.$$

(a) (6 points) Find the indicial equation corresponding to the regular singular point $x_0 = 0$. (Do not try to solve the differential equation).

(b) (6 points) Write the general form of the solution(s).

5. Consider the function $f(x) = |x|$ on $[-\pi, \pi]$ and $f(x + 2\pi) = f(x)$.
   (a) (2 points) What is the period $2L$ of $f(x)$?

   (b) (2 points) Is $f(x)$ an odd or even function? Show your work.

   (c) (5 points) Find the sine terms of the Fourier series of $f(x)$.

   (d) (5 points) Find the cosine terms (including a_0) of the Fourier series of $f(x)$.

(e) (4 points) Write the Fourier series $F(x)$ of the function $f(x)$.

(f) (3 points) Using Fourier series convergence theorem, check the points where $F(x)$ and $f(x)$ agree and do not agree.

(g) (3 points) Use part (e) and verify that

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}.$$

SCRATCH PAPER