UCONN - Math 3410 - Fall 2016 - Quiz 1

Name: Solution Key

1. [**5 points**] Find all value(s) of α and β for which the following differential equation becomes exact.
$$4x^3 - \alpha x^2 y + y^3 - 2\beta + (\beta x^3 + 3xy^2)y' = 0$$
Solution: If we rewrite the DE, then we get
$$(4x^3 - \alpha x^2 y + y^3 - 2\beta)dx + (\beta x^3 + 3xy^2)dy = 0.$$
In our notation $M = 4x^3 - \alpha x^2 y + y^3 - 2\beta$ and $N = \beta x^3 + 3xy^2$. Since above DE is exact then we have $M_y = N_x$;
$$M_y = -\alpha x^2 + 3y^2 = N_x = 3\beta x^2 + 3y^2.$$
By solving this identity one gets $-\alpha = 3\beta$. Therefore, for any values of α, β with $-\alpha = 3\beta$ the above DE becomes exact. Equivalently, $\alpha = 3t$ and $\beta = -t$ for every t.

2. [**5 points**] Solve the following differential equation with the give initial value.
Solution.
$$y' = \frac{xe^{2x}}{y}, \qquad y(0) = \frac{1}{2}.$$
One should realize that this DE is separable;
$$ydy = xe^{2x}dx$$
by integrating both sides one gets;
$$\int ydy = \int xe^{2x}dx \quad \text{equaivalently} \quad \frac{y^2}{2} = \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + c.$$
Use the initial value to find c;
$$\frac{1}{8} = 0 - \frac{1}{4} + c \quad \text{so} \quad c = \frac{3}{8}.$$
Hence the solution is
$$\frac{y^2}{2} = \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + \frac{3}{8}.$$