

# UConn - Math 3410 - Fall 2017 - Quiz 4

Name: Solution KEY

**Question:** Solve the first order equation

$$3u_x + 2u_y = 0$$

with the auxiliary condition

$$u(x, 0) = \sin(x).$$

**Solution:** Notice that

$$\langle (3, 2), \nabla u(x, y) \rangle = 3u_x + 2u_y = 0$$

Hence  $u(x, y)$  is constant in the direction of  $(3, 2)$ . The lines parallel to  $(3, 2)$  have equations  $2x - 3y = 0$ . here  $2x - 3y = 0$  is called the characteristic lines. As  $u(x, y)$  is constant on these lines therefore  $u(x, y)$  depends only  $2x - 3y$ . Hence

$$u(x, y) = f(2x - 3y).$$

Using the the auxiliary condition  $u(x, 0) = \sin(x)$  we get

$$u(x, 0) = f(2x) = \sin(x).$$

Since  $f(2x) = \sin(x)$  we can find  $f(x) = \sin(x/2)$ . Since

$$u(x, y) = f(2x - 3y) = \sin((2x - 3y)/2)$$

is the solution. (you can check you answer by finding  $u_x$  and  $u_y$  and verify that  $3u_x + 2u_y = 0$ .)