UCONN - Math 3410 - Fall 2017 - Solution to Graded Problems of HW2

Question 1 (Problem 2, 4 Points) *Check that if the differential equation is exact*

$$e^x(x+1)dx + (ye^y - xe^x)dy = 0$$

and if it is exact then solve the differential equation.

Solution: Since

$$M = e^x(x+1) \quad \text{and} \quad N = ye^y - xe^x$$

and

$$M_y = 0 \quad \text{whereas} \quad N_x = -e^x - xe^x.$$

Therefore, the DE is not exact. To find the solution they need to check that

$$\frac{M_y - N_x}{N} = \text{independet of } x \quad \text{or} \quad \frac{M_y - N_x}{M} = \text{independet of } y$$

The second one holds;

$$\frac{M_y - N_x}{M} = \frac{-e^x - xe^x}{e^x(x+1)} = -1.$$

Then

$$\mu(x) = e^{-\int \frac{M_y - N_x}{M} dy} = e^{\int dy} = e^y$$

is an integrating factor. Now one needs to multiply the DE with the integrating factor

$$e^y e^x(x+1)dx + e^y(ye^y - xe^x)dy = 0$$

to get an exact DE. Now above DE is exact, then the general solution is

$$0 = f(x,y) = \int e^y e^x(x+1)dx = e^y(xe^x - e^x) + g(y).$$

Use $f_y = e^y(ye^y - xe^x)$ to find g which gives

$$f(x,y) = 2xe^{x-y} + y^2 = c$$

is general solution to the DE.

Question 2 (Problem 3, 4 points) *Let $P(x) = \int p(x)dx$. Show that $e^{P(x)}$ is an integrating factor for the DE*

$$y' + p(x)y = q(x).$$

Solution: Since the integrating factor is given we can first rewrite the DE as

$$dy + (p(x)y - q(x))dx = 0$$

Then multiplying DE with the integrating factor one gets

$$e^{P(x)}dy + e^{P(x)}(p(x)y - q(x))dx = 0.$$

Now one needs to show that this DE is exact. To do this, using our standard notation, we first write

$$M(x,y) = e^{P(x)}(p(x)y - q(x)) \quad \text{and} \quad N(x,y) = e^{P(x)}$$

Then as

$$M_y = e^{P(x)}p(x) \quad \text{and} \quad N_x = e^{P(x)}\frac{d\,P(x)}{dx} = e^{P(x)}p(x).$$

are equal to each other, one conclude that above DE is exact and $e^{P(x)}$ is an integrating factor.