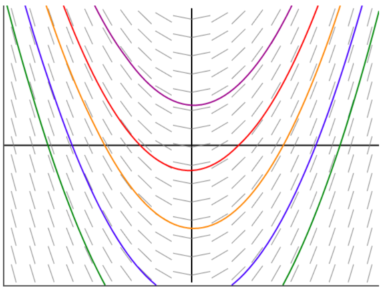
UCONN - Math 3410 - Fall 2017 - Supplementary problems for Exam 1

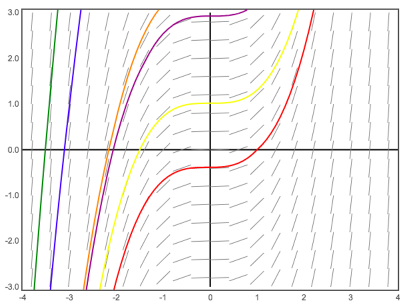
Question 1 *Sketch the gradient field for the following DEs, sketch at least three solution curves, and sketch the solution passing through* $(1,0)$.

1. $y' = x$.
2. $y' = x^2$.
3. $y' = x + y$.
4. $y' = 2$.
5. $y' = e^x$.

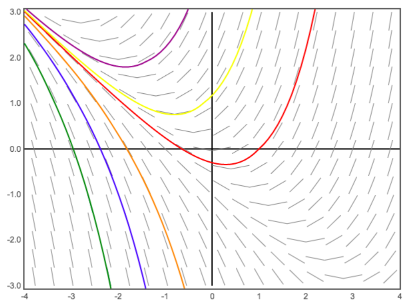
Answer:



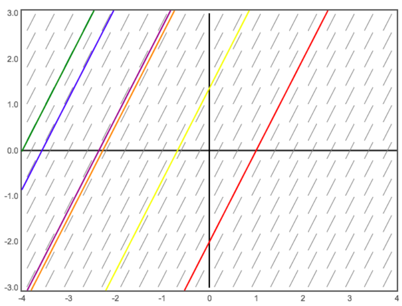
(a) ...



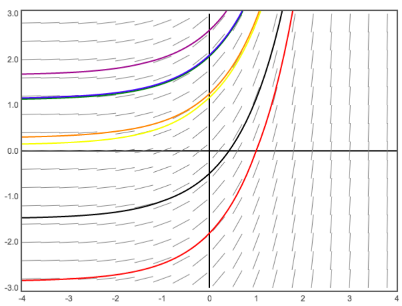
(b) ...



(c) ...



(d) ...



(e) ...

Question 2 *Show that* $y = e^{4x}$ *is a solution to the DE*

$$y' - 4y = 0.$$

Question 3 *Show that* $y = c_1 e^{2x}\cos(x) + c_2 e^{2x}\sin(x)$ *is a 2-parameter family of solutions to the DE*

$$y'' - 4y' + 5y = 0.$$

Thanks for Michael for the correction that there is $\cos(x)$ *there now.*

Question 4 *Find the order of the DEs and write if they are linear or nonlinear equations.*

1. $y^{(7)} + 25y^{(16)} + y'' + y = 0$.
2. $y''' + 2y'(y'')^2 + y' = 0$.
3. $y'' - y' = 0$.

Question 5 *Show that* $\frac{y^3}{3} - \frac{x^2}{2} + 5x = 0$ *is an implicit solution to the DE*

$$y^2y' = (x-5).$$

Question 6 *Show that* $x^2y^2 + x = 1$ *is an implicit solution to the DE*

$$2x^2yy' + 2xy^2 + 1 = 0.$$

Question 7 *Without solving the DE*

$$(x^2-9)y' + 2y = \ln|20-4x|,$$

determine the interval of validity of the solution.

Also fin the largest interval for which the initial value $y(0) = 0$ *has a unique solution.*

Question 8 *Solve the following DEs*

1. $y' = xye^x$.
2. $y' = e^{x+y}$.
3. $2x + 2yy' = 0$.
4. $y' + 2x(y+1) = 0$ *with* $y(0) = 2$.

Question 9 *Check if the DE is exact*

$$(x^2-y)dy + (2x^3+2xy)dx = 0.$$

Then solve the DE.

Question 10 *Check if the DE is exact*

$$(2x-1)(y-1)dx + (x+2)(x-3)dy = 0.$$

Then solve the DE.

Question 11 *Check if the DE is exact*

$$(3x^2y^2 - 4xy)y' = 2y^2 - 2xy^3.$$

Then solve the DE.

Question 12 *Solve the following Bernoulli's equation*

$$xy' + y + x^2y^2e^x = 0.$$

Answer: $y = \frac{1}{x(c_1+e^x)}$.

Question 13 *Solve the following Bernoulli's equation*

$$xyy' = y^2 - x^2.$$

Answer: $y^2 = -2x^2 \ln x + c_1 x^2$.

Question 14 *Solve the following Bernoulli's equation*

$$y' + \frac{4}{x}y = x^3 y^2 \quad \textit{with} \quad y(2) = -1,\ x > 0.$$

Answer: $y = \frac{16}{x^4(16\ln 2 - 16\ln x - 1)}$.

Question 15 *Solve the following Bernoulli's equation*

$$6y' - 2y = xy^4 \quad \textit{with} \quad y(0) = -2.$$

Answer: $y = \frac{2e^{\frac{x}{3}}(-4(xe^x - e^x) - 5)^{\frac{2}{3}}}{-4xe^x + 4e^x - 5}$.

Question 16 *Solve the following Ricatti's equation*

$$y' = y^2 + 2xy + (x^2 - 1) \quad \textit{with given particular solution} \quad y_1(x) = -x.$$

Answer: $y = -x + \frac{1}{c_1 - x}$.

Question 17 *For the following autonomous differential equation*

$$y' = (y+2)^2(1-y)^2$$

(a) *Find and classify all of its equilibrium solutions.*

(b) *If* $y(-2) = 1$ *then what is* $\lim_{x\to\infty} y(t)$? *and what is* $\lim_{x\to-\infty} y(t)$*?*

(c) *If* $y(1) = 0$ *then what is* $\lim_{x\to\infty} y(t)$? *and what is* $\lim_{x\to-\infty} y(t)$*?*

(d) *If* $y(3410) = \alpha$ *and* $\lim_{x\to\infty} y(t) = 1$, *find all possible values of* α.

Question 18 *For the following autonomous differential equation*

$$y' = -y^2(y-2)^3(y+2)$$

(a) *Find and classify all of its equilibrium solutions.*

(b) *If* $y(-2) = 1$ *then what is* $\lim_{x\to\infty} y(t)$? *and what is* $\lim_{x\to-\infty} y(t)$*?*

(c) *If* $y(1) = 0$ *then what is* $\lim_{x\to\infty} y(t)$? *and what is* $\lim_{x\to-\infty} y(t)$*?*

(d) *If* $y(3410) = \alpha$ *and* $\lim_{x\to\infty} y(t) = 2$, *find all possible values of* α.

Question 19 *Suppose that* $y_1(x)$ *and* $y_2(x)$ *are two linearly independent solutions to the following DE*

$$x^2 y'' + 2x^3 y' - sinxy = 0.$$

Using Abel's theorem, find the Wronskian of y_1 *and* y_2, $W(y_1, y_2)$.

Question 20 *Find the orthogonal trajectories of the family of straight lines through* $(1, 0)$.

Question 21 *Find the orthogonal trajectories of the family of parabolas* $y = ax^2$.

Question 22 *Find the orthogonal trajectories of the family of ellipsoid* $2x^2 + y^2 = r^2$.

Question 23 *Find the Laplace transform of the following functions*

1. $f(x) = e^{-x}(x^2 + 5x - 5)$.
2. $f(x) = \begin{cases} 2x & 0 \leq x < 1 \\ 1 & 1 < x < \infty. \end{cases}$

Question 24 *Suppose that* $\mathcal{L}\{f(x)\} = \frac{s^2}{s^2+1}$ *for some function* $f(x)$ *with given that* $f(0) = -1$, $f'(0) = 2$, *and* $f''(0) = 1$.

1. *Find* $\mathcal{L}\{xf(x)\}$.
2. *Find* $\mathcal{L}\{x^2 f(x)\}$.
3. *Find* $\mathcal{L}\{e^{-2x} f(x)\}$.
4. *Find* $\mathcal{L}\{x^2 e^{-2x} f(x)\}$.
5. *Find* $\mathcal{L}\{f'(x)\}$.
6. *Find* $\mathcal{L}\{f''(x)\}$.
7. *Find* $\mathcal{L}\{xf''(x)\}$.

Question 25 *Find the inverse Laplace transform of the following functions*

1. $F(s) = \frac{3}{s^2}$.
2. $F(s) = \frac{s+1}{s(s+2)}$.
3. $F(s) = \frac{s^2+1}{s^2(s+2)}$.

Question 26 *It can be shown that for a constant* $a \neq 0$

$$\mathcal{L}\{\cos ax\} = \frac{s}{s^2 + a^2} \quad s > 0$$

and

$$\mathcal{L}\{\sin ax\} = \frac{a}{s^2 + a^2} \quad s > 0.$$

Using this answer the following questions.

1. *Find* $\mathcal{L}\{x \sin ax\}$ *and* $\mathcal{L}\{e^{-3x} \sin ax\}$.
2. *Find* $\mathcal{L}^{-1}\{\frac{2}{s^2+9}\}$ *and* $\mathcal{L}^{-1}\{\frac{1}{s^3} + \frac{2}{s^2+4}\}$.

Question 27 *Use the Laplace transform to solve the following initial value problem*

$$y'' - 2y' + 2y = e^{-x} \quad \text{with } y(0) = 0 \text{ and } y'(0) = 1.$$

Question 28 *Use the Laplace transform to solve the following initial value problem*

$$y'' + y = \cos 2x \quad \text{with } y(0) = 2 \text{ and } y'(0) = 1.$$