Spring 2018 - Math 3435
Exam 1 - February 21
Time Limit: 50 Minutes

Name (Print): ____________________

This exam contains 6 pages (including this cover page) an empty scratch paper and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books or notes on this exam.

You are required to show your work on each problem on this exam.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

[1] Exam template credit: `http://www-math.mit.edu/~psh`

1. Consider the following two partial differential equations(PDEs)

$$(\star)\ \ L[u] = u_{xx} + u_{yy} = 0 \quad \text{and} \quad (\star\star)\ \ L[u] = u_t + 6uu_x - u_{xxx} = 0.$$

   (a) (4 points) Find the order of the PDE

   PDE in $(\star)$ has order .... and PDE in $(\star\star)$ has order ....

   (b) (3 points) Show that if the PDEs are **linear** or **non-linear** [Show your work]

   PDE in $(\star)$ is .... and PDE in $(\star\star)$ is ....

   (c) (3 points) Consider $u_1(x,y) = x^3 + x^2 - y^2$ which solves $u_{xx} + u_{yy} = 6x$ and $u_2(x,y) = y^2 + 2x$ which solves $u_{xx} + u_{yy} = 2$. Can you find a function u which solves $u_{xx} + u_{yy} = 3x + 4$? [Show your work].

2. [Problems 1 and 3 from HW2]

   (a) (6 points) Using ODE techniques find the general solutions of the following PDE for $u = u(x, y)$

   $$u_x + 2xu = 4xy.$$

   (b) (4 points) For the PDE in part (a), find a particular solution satisfying the side condition

   $$u(x, x) = 0.$$

3. (10 points) Solve the following first order constant coefficient PDE

$$4u_x - 3u_y = 0 \quad \text{subject to} \quad u(0, y) = y + y^3$$

4. [Problems 8 and 9 from HW3]

   (a) (6 points) Obtain the general solution of the following PDE

$$yu_x - 4xu_y = 2xy \quad \text{for all } (x, y).$$

   (b) (4 points) Find the particular solution of the PDE you found in (a) satisfying the side condition

$$u(x, 0) = x^4.$$

5. Consider the following PDE

$$\begin{cases} u_t = u_{xx}, \quad 0 < x < 2, \quad t > 0, & \text{The Heat equation} \\ u(0,t) = 0 \quad \text{and} \quad u(2,t) = 0, \quad t > 0, & \text{The boundary conditions} \\ u(x,0) = 3\sin(\pi x) - 4\sin(\frac{3\pi x}{2}), \quad 0 < x < 2 & \text{The initial condition.} \end{cases}$$

(a) (3 points) Which of the following solves the given heat equation with boundary conditions and initial condition [You should find the correct answer without solving the PDE].

1. $u(x,t) = 4e^{-\frac{2^2\pi^2}{2^2}t} - 3e^{-\frac{3^2\pi^2}{2^2}t}$.
2. $u(x,t) = 3\sin(\pi x) - 4\sin(\frac{3\pi x}{2})$.
3. $u(x,t) = 3\cos(\pi x)e^{-\frac{2^2\pi^2}{2^2}t} - 4\cos(\frac{3\pi x}{2})e^{-\frac{3^2\pi^2}{2^2}t}$.
4. $u(x,t) = 3\sin(\pi x)e^{-\frac{2^2\pi^2}{2^2}t} - 4\sin(\frac{3\pi x}{2})e^{-\frac{3^2\pi^2}{2^2}t}$.

(b) (3 points) Is the solution you found in part (a) the only solution? Can there be any other solutions to the above PDE?

(c) (4 points) Verify that the solution to above PDE satisfies $-7 \leq u(x,t) \leq 7$ for $0 \leq x \leq 2$ and $t \geq 0$.

(d) [Bonus(5 points)] If we replace the initial condition with $u(x,0) = 0$, without solving the Heat equation, can you find the solution explicitly?