Spring 2018 - Math 3435
Practice Exam 1 - February 21
Time Limit: 50 Minutes

Name (Print): ______________________________

This exam contains 6 pages (including this cover page) an empty scratch paper and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books or notes on this exam.

You are required to show your work on each problem on this exam.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1

[1]Exam template credit: `http://www-math.mit.edu/~psh`

1. Consider the following two partial differential equations(PDEs)

$$(\star)\ \ L[u] = u_t - u_{xx} = 0 \quad \text{and} \quad (\star\star)\ \ L[u] = u - u_{xxt} + uu_{tt} = 0.$$

(a) (4 points) Find the order of the PDE

PDE in $(\star)$ has order .... and PDE in $(\star\star)$ has order ....

(b) (3 points) Show that if the PDEs are **linear** or **non-linear** [Show your work]

PDE in $(\star)$ is .... and PDE in $(\star\star)$ is ....

(c) (3 points) Suppose that u_1 solves $u_t - u_{xx} = f(x,t)$ and u_2 solves $u_t - u_{xx} = g(x,t)$ for some f, g. Can you find a function u which solves $3435f(x,t) + 2018g(x,t)$? [Show your work].

2. (a) (6 points) Using ODE techniques find the general solutions of the following PDE for $u = u(x, y)$

$$yu_{xy} + 2u_x = x.$$

(b) (4 points) For the PDE in part (a), find a particular solution satisfying the side condition

$$u(x, 1) = 0 \quad \text{and} \quad u(0, y) = 0.$$

3. (10 points) Solve the following PDE

$$u_x - 2u_y = 0 \quad \text{subject to} \quad u(x, e^x) = e^{2x} + 4xe^x + 4x^2$$

4. (a) (6 points) Find the general solution of the following PDE

$$xu_x - xyu_y = 0 \quad \text{for all } (x, y).$$

(b) (4 points) Find the particular solution of the PDE you found in (a) satisfying the side condition

$$u(x, x) = x^2 e^{2x}.$$

5. Consider the following PDE

$$\begin{cases} u_t - 9u_{xx} = 0 \quad \text{whenever } 0 \leq x \leq 3,\ t \geq 0, \\ u(0,t) = 0 \quad \text{and} \quad u(3,t) = 0, \\ u(x,0) = 1417\sin(\pi x) + 2018\sin(3\pi x). \end{cases}$$

(a) (3 points) Which of the following solves the given heat equation [You should find the correct answer without solving the PDE].

1. $u(x,t) = 2018e^{-81t}\sin(\pi x) + 1417e^{-9^3 t}\sin(3\pi x)$.
2. $u(x,t) = 1417\sin(\pi x) + 2018\sin(3\pi x)$.
3. $u(x,t) = 1417e^{-81t}\sin(\pi x) + 2018e^{-9^3 t}\sin(3\pi x)$.
4. $u(x,t) = 1417e^{-81t}\cos(\pi x) + 2018e^{-9^3 t}\cos(3\pi x)$.

(b) (3 points) Is the solution you found in part (a) the only solution? Can there be any other solutions to the above PDE?

(c) (4 points) Verify that the solution to above PDE satisfies $-3435 \leq u(x,t) \leq 3435$ for $0 \leq x \leq 3$ and $t \geq 0$.