Spring 2018 - Math 3435
Practice Exam 2 - March 28
Time Limit: 50 Minutes

This exam contains 6 pages (including this cover page) an empty scratch paper and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may not use your books or notes on this exam.

You are required to show your work on each problem on this exam.

Do not write in the table to the right.

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<th>Problem</th>
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1Exam template credit: http://www-math.mit.edu/~psh
1. Let

\[ f(x) = \begin{cases} 
1 & \text{when } 0 \leq x \leq L, \\
0 & \text{when } -L \leq x < 0.
\end{cases} \]

(a) (5 points) Find the Fourier series \( \mathcal{F}(x) \) of \( f(x) \) on \([-L, L]\).

(b) (2 points) At which points on \([-L, L]\), do \( \mathcal{F}(x) \) and \( f(x) \) NOT agree?

(c) (3 points) Verify that

\[ \frac{\pi}{4} = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}. \]
2. (10 points) Solve

\[
\begin{align*}
    u_t - u_{xx} &= e^{-4t} \cos(t) \sin(2x), \quad 0 \leq x \leq \pi, \quad t \geq 0, \\
    u(0, t) &= 0, \quad u(\pi, t) = 0, \\
    u(x, 0) &= \sin(3x) .
\end{align*}
\]
3. (10 points) Consider the Heat equation

\[
\begin{aligned}
& u_t - 2u_{xx} = 0, & 0 \leq x \leq 1, & t \geq 0, \\
& u(0, t) = -1, & u_x(1, t) = 1, \\
& u(x, 0) = x + \sin\left(\frac{3\pi x}{2}\right) - 1.
\end{aligned}
\] (2)

where the boundary conditions are non-homogeneous. Transform the equation into a new one with homogeneous boundary conditions. (You do not need to solve the new equation).
4. (10 points) Describe the steps how to solve the following heat equation

\[
\begin{align*}
&u_t - ku_{xx} = h(x, t), \quad 0 \leq x \leq L, \ t \geq 0, \\
&u(0, t) = a(t), \ u(L, t) = b(t), \\
&u(x, 0) = f(x).
\end{align*}
\]
5. (5 points (bonus)) Let \( u \) be a solution to the following heat equation

\[
\begin{align*}
&\left\{ \begin{array}{ll}
& u_t = u_{xx}, \\
& u_x(0,t) = 0, \quad u_x(\pi, t) = 0, \quad t \geq 0, \\
& u(x,0) = f(x), \quad 0 < x < \pi.
\end{array} \right. \\
\end{align*}
\]

Show that

\[
I(t) = \int_0^\pi e^{u(x,t)} \, dx
\]

decreases as a function of \( t \) for \( t \geq 0 \).