UCONN
UNIVERSITY OF CONNECTICUT

Spring 2018 - Math 3435   **Name (Print):** _____
Practice Exam 2 - March 28
Time Limit: 50 Minutes

This exam contains 6 pages (including this cover page) an empty scratch paper and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books or notes on this exam.

You are required to show your work on each problem on this exam.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	0	
Total:	40	

1. Let
$$f(x) = \begin{cases} 1 & \text{when } 0 \leq x \leq L, \\ 0 & \text{when } -L \leq x < 0. \end{cases}$$

(a) (5 points) Find the Fourier series $\mathcal{F}(x)$ of $f(x)$ on $[-L, L]$

(b) (2 points) At which points on $[-L, L]$, do $\mathcal{F}(x)$ and $f(x)$ **NOT** agree ?

(c) (3 points) Verify that
$$\frac{\pi}{4} = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}.$$

2. (10 points) Solve

$$\begin{cases} u_t - u_{xx} = e^{-4t} \cos(t) \sin(2x), & 0 \le x \le \pi, \ t \ge 0, \\ u(0,t) = 0, \ u(\pi,t) = 0, \\ u(x,0) = \sin(3x). \end{cases} \tag{1}$$

3. (10 points) Consider the Heat equation

$$\begin{cases} u_t - 2u_{xx} = 0, & 0 \leq x \leq 1, \ t \geq 0, \\ u(0,t) = -1, \ u_x(1,t) = 1, \\ u(x,0) = x + \sin(\frac{3\pi x}{2}) - 1. \end{cases} \tag{2}$$

where the boundary conditions are **non-homogeneous**. Transform the equation into a new one with **homogeneous** boundary conditions. (You do not need to solve the new equation).

4. (10 points) Describe the steps how to solve the following heat equation

$$\begin{cases} u_t - ku_{xx} = h(x,t), & 0 \leq x \leq L, \ t \geq 0, \\ u(0,t) = a(t), \ u(L,t) = b(t), \\ u(x,0) = f(x). \end{cases} \tag{3}$$

5. (5 points (bonus)) Let u be a solution to the following heat equation

$$\begin{cases} u_t = u_{xx}, & 0 \leq x \leq \pi, \ t \geq 0, \\ u_x(0,t) = 0, \ u_x(\pi,t) = 0, & t \geq 0 \\ u(x,0) = f(x), & 0 < x < \pi. \end{cases}$$

Show that

$$I(t) = \int_0^\pi e^{u(x,t)} dx$$

decreases as a function of t for $t \geq 0$.