UNIVERSITY OF CONNECTICUT

Spring 2018 - Math 3435
Practice Final Exam - April 30
Time Limit: 120 Minutes

Name (Print): ____________________

This exam contains 9 pages (including this cover page) an empty scratch paper and 8 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books or notes on this exam.

You are required to show your work on each problem on this exam.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Total:	80	

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[1] Exam template credit: `http://www-math.mit.edu/~psh`

1. (10 points) Find general solution of the following PDE for $u = u(x, y)$ using ODE techniques.

$$u_x + 2xu = 4xy.$$

satisfying the side condition $u(x, x) = 0$.

2. (10 points) Let u be a solution to the following heat equation

$$\begin{cases} u_t = u_{xx}, & 0 < x < \pi,\ t > 0, \\ u_x(0,t) = 0 = u_x(\pi,t), & t \geq 0 \\ u(x,0) = f(x), & 0 \leq x \leq \pi. \end{cases}$$

Show that

$$I(t) = \int_0^{\pi} e^{u(x,t)} dx$$

decreases as a function of t for $t \geq 0$.

3. (10 points) Solve the Poisson equation

$$\begin{cases} u_{xx} + u_{yy} = x^2 + y^2, & x^2 + y^2 < 4, \\ u(x,y) = 0 & x^2 + y^2 = 4. \end{cases}$$

using the polar coordinates. You may want to look for solutions of the form $u(x,y) = f(x^2 + y^2)$.

4. (10 points) Suppose that$u(x,t)$ is solution of the diffusion equation with variable dissipation

$$\begin{cases} u_t - ku_{xx} + h(t)u = 0 & -\infty < x < \infty,\ t \geq 0, \\ u(x,0) = f(x) & -\infty < x < \infty. \end{cases}$$

and $g(t)$ is a solution to $g'(t) = h(t)g(t)$ with $g(0) = 1$. Then show that $v(x,t) = g(t)u(x,t)$ is a solution of

$$\begin{cases} v_t - kv_{xx} = 0 & -\infty < x < \infty,\ t \geq 0, \\ v(x,0) = f(x) & -\infty < x < \infty. \end{cases}$$

5. Let $U(r,\theta)$ be the solution to the following Dirichlet problem

$$\begin{cases} U_{rr} + \frac{1}{r}U_r + \frac{1}{r^2}U_{\theta\theta} = 0 & \text{in the disk with radius } r < 2, \\ U(2,\theta) = 1 + 3\sin(2\theta). \end{cases}$$

Answer the following questions.

(a) (4 points) Write the solution using Poisson's integral formula.

(b) (4 points) Find the maximum value of $U(r,\theta)$ in the disk with radius $r \leq 2$.

(c) (3 points) Calculate the value of $U(r,\theta)$ at the origin.

6. (10 points) Find the solution $u(x,t)$ to the following Wave equation

$$\begin{cases} u_{tt} - u_{xx} = x - t & -\infty < x < \infty,\ -\infty < t < \infty, \\ u(x,0) = x^2 \text{ and } u_t(x,0) = \sin(x), & -\infty < x < \infty. \end{cases}$$

7. Let $u(x,t)$ be the solution to the following heat equation

$$\begin{cases} u_t - 8u_{xx} = \cos(t) + e^t \sin(x/2) & 0 \le x \le \pi,\ t \ge 0 \\ u(0,t) = \sin(t),\ \ u_x(\pi,t) = 0 & t \ge 0, \\ u(x,0) = 0. & \end{cases}$$

(a) (3 points) Find a particular solution u_p and let $v(x,t) = u(x,t) - u_p(x,t)$ so that $v(x,t)$ satisfies the homogeneous boundary condition and solves the non-homogeneous heat equation.

(b) (3 points) Write the PDE for which $v(x,t)$ solves, the boundary conditions and the initial condition $v(x,t)$ satisfies.

(c) (4 points) Without solving the new equation corresponding to v describe how to solve it.

8. Let $f(x)$ be given as

$$f(x) = x \quad \text{when} \quad 0 \leq x < \pi.$$

(a) (5 points) Extend $f(x)$ into an even periodic function with period of 2π.

(b) (5 points) Find Fourier series $F(x)$ of the function you found in (a).