Maxwell's Equations

James Clerk Maxwell 1831—1879.

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$\varepsilon_0 = 8.854 \times 10^{-12}$ permittivity constant

$\nabla = \left(\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n}\right)$

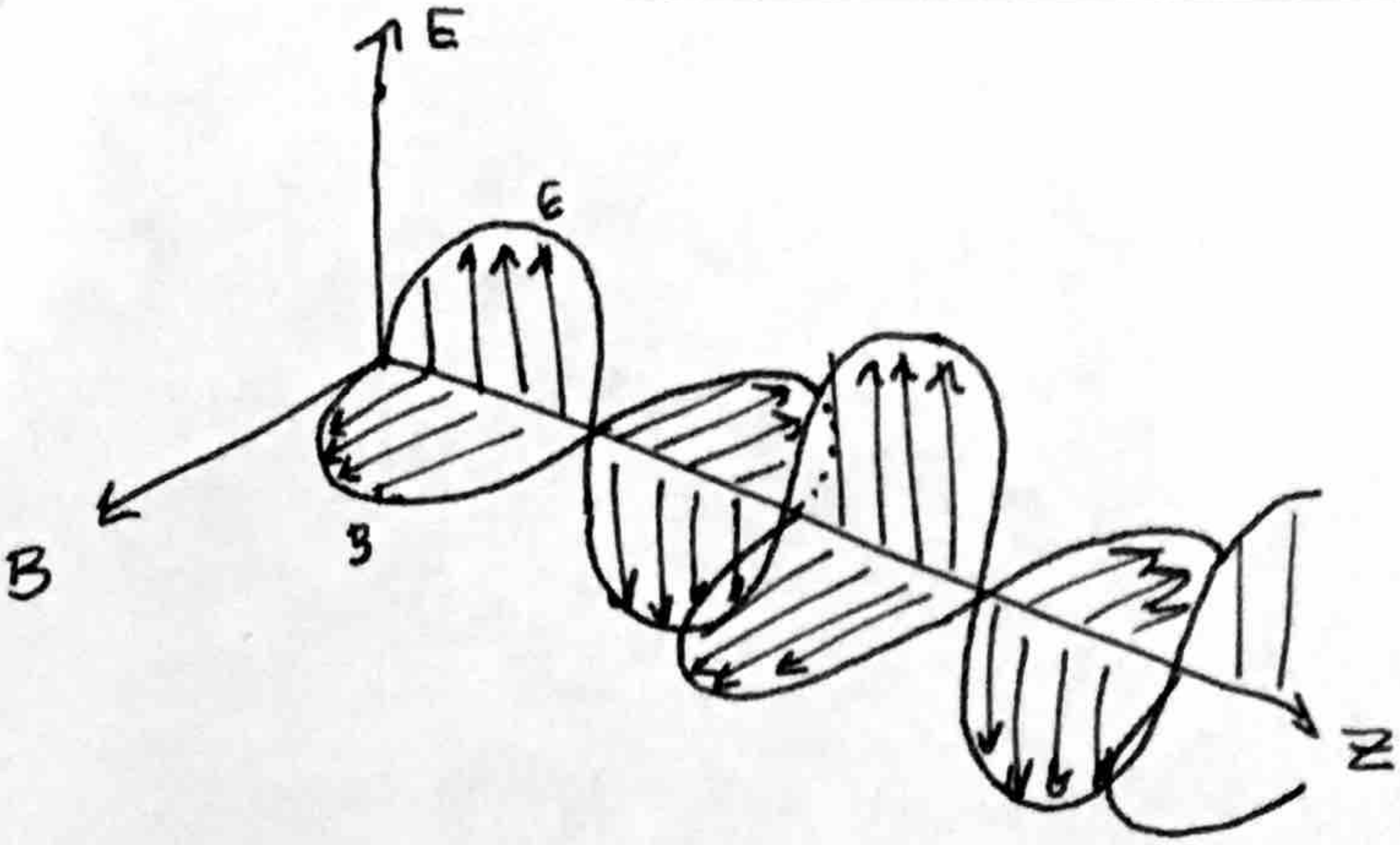
$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

Set of 4 basic PDE that form the foundation for our understanding of classical Electromagnetism.

- Electric/magnetic fields are related
  - generated by changes and currents.
  - Demonstrate how electric and magnetic fields propagate at speed of light



· Vector Calculus has simplified these equations.

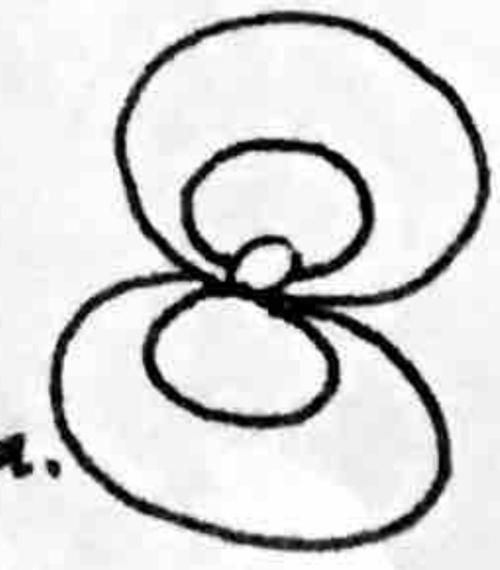
<u>Gauss' Law</u> $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$ Electric flux leaving a volume is prop. to charge inside.

Have an electric charge sitting in a Gaussian surface, relates the electric flux to charge.

<u>Gauss's Law Magnetism</u>

$$\nabla \cdot \vec{B} = 0$$

· Essentially no magnetic monopoles.
· Magnetic fields only come in dipoles.
· Fild lines form infinite loops.
· Magnetic field enters a volume must exit a volume, vise-versa.



· Magnetic flux through any Gaussian surface is zero

Gaussian Laws are about flux.

How the flow of electric or magnetic fields through a surface or volume ~~affects the above~~ is related to changes inside.

- Next ~~to~~ 2 equations ~~the~~ have to do w/ the phenomena of electromagnetic induction
- How electric/magnetic ~~flux~~ fields affect ~~electric and mag~~ each other as they move with time.

Faraday's Law of Induction (1831)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

describes how varying magnetic field creates, or "induces" magnetic field

This basic ~~form~~ idea of electromagnetic radiation can be seen in simple electric generators

– general principle

rotating bar ~~magnets~~ magnet, "generates", induces electric field in wire.

Ampère's Law (with Maxwell's addition)

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

$\vec{J}$ = total current per unit area.

- States essentially the opposite.

1.) Magnetic fields can be induces by electric current

André-Marie Amphere (1823) discovered

- relates integrated mag. field around a closed loop to the electric current passing through a loop.

2.) Maxwell's addition

Integ. magnetic fields around a closed loop can be induced by changing electric ~~around~~ fields

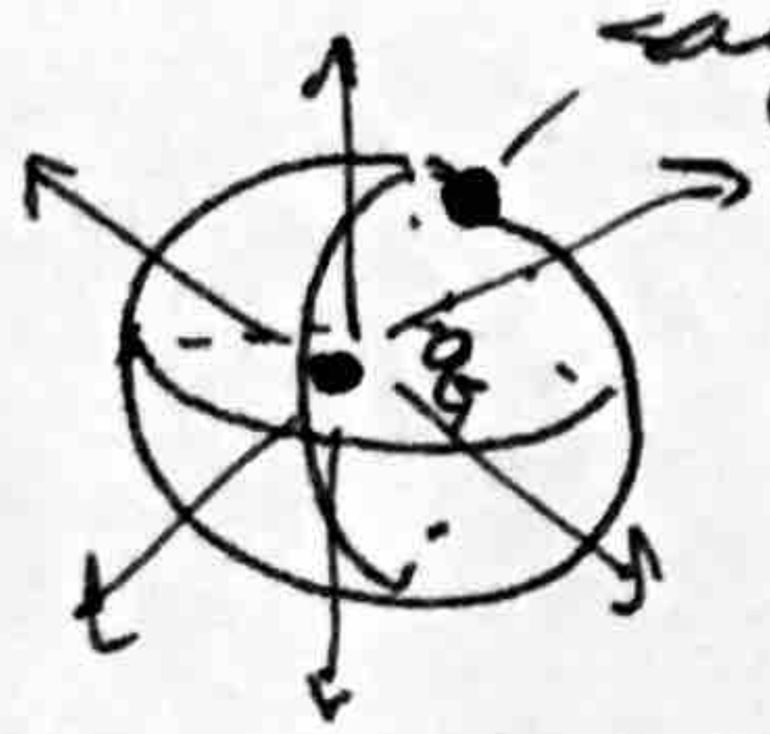
- This makes it mathematically consistent for non-static fields.

Example

Gauss's Law

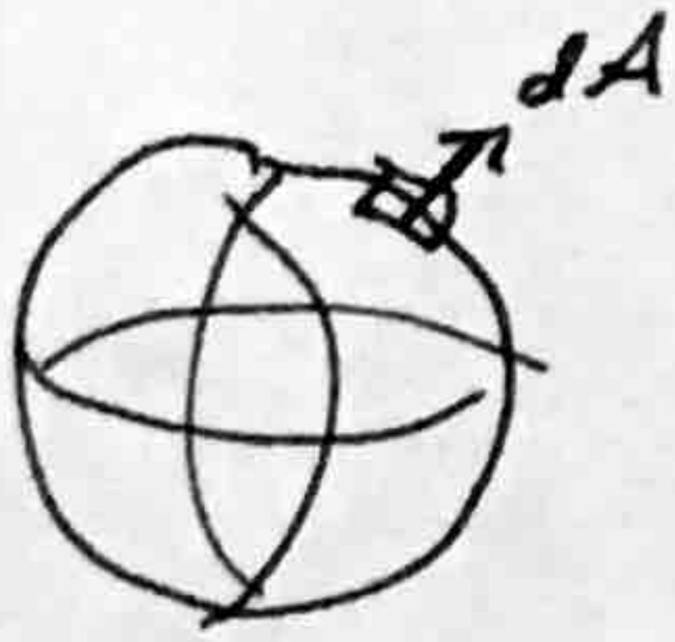
$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\Phi_E = \oint_S E \cdot dA = \frac{\rho \text{(charge enclosed)}}{\varepsilon_0}$$



say we want to know electric field here.

pick Gaussian surface.



dA ~~are~~ are alway pointing in same direction, parralell

$$\cancel{E \oint dA = \frac{\rho}{\varepsilon_0}}$$

$$\oint E\, dA = \frac{\rho}{\varepsilon_0}$$

↓ dot product goes away

$$E(4\pi r^2) = \frac{\rho}{\varepsilon_0}$$

$$E = \frac{1\rho}{4\pi r^2 \varepsilon_0}$$

coulomb's constant

$$E = \frac{k\rho}{r^2}$$