Poisson's equation

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What is Poisson’s Equation?

The Generalized Equation and Basic Properties

$$\Delta\varphi = f$$

This is a second order PDE that is elliptic, and Poisson’s Equation is usually used to as a common example for PDE’s of this type as it is relatively forward.

Terms and Functions

Laplace Operator

Our initial term represents the Laplace operator.

In where we normally dwell, the 3 Dimensional Euclidean space, this operator is typically rewritten as the gradient multiplied be the divergence

$$\nabla^2$$

This can be further broken down to rewrite Poisson's Equation in its most familiar form:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\varphi(x, y, z) = f(x, y, z)$$

This operator as we can see represents the slope of the tangent of our function squared, and happens to naturally describes several scientific fields.

$$\Delta$$

Functions

Both of these functions can be real or complex-valued, showing the potential versatility of this equation.

Typically, f is the given function, and φ is the sought-after function.

When f is 0, we actually have Laplace’s equation.

The relationship of these function is that one is a projection of the other on a vector field.

Due to its similarity to Laplace’s Equation, Poisson’s Equation can also be solved by Green’s Function.

$$\varphi \qquad f$$

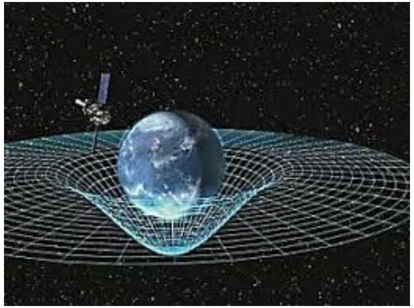
$$\Delta\varphi = f$$

About Poisson

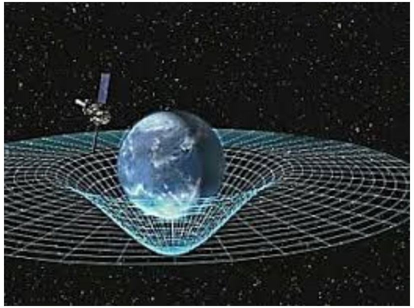
Siméon Poisson

Born in France in 1781, Poisson was recognized as a talented mathemeticion at age 17 as soon as he entered high level schooling. In two years of studying he published two books on mathematical ideas. After this publishing, he was able to grow close to other famed mathematicians such as Pierre-Simon Laplace, who he worked very closely with. His contributions to various fields of math is impressive as his Poisson's Equation is very widely used today. Some of his other contributions are his equation for surface electric potential at a particular point, and the distribution of charge in a fluid in the Poisson-Boltzmann Equation. For those who are taking Elementary Stochastic Processes, you may recognize the Poisson Distribution $P(k \text{ events in interval}) = e^{-\lambda}\frac{\lambda^k}{k!}$. On top of this he published many memoirs describing all of his above works. With his talent he has contributed greatly to many forms of math and physics and his works have inspired other famed mathematicians such as Lagrange, Lacroix and Legendre.

Uses for Poisson’s Equation



Newtonian gravity



Newton's Law of universal gravitation states that a particle attracts every other particle in the universe with a force that is directly proportional to the product of their masses, and inversely proportional to the square of the distance of their centers.

$$F = G\frac{m_1 m_2}{r^2}$$

This is perhaps the most common and famous use of Poisson's Equation, Newtonian Gravity is defined by the Poisson's Equation

$\nabla^2 \phi = 4\pi G \rho.$ where $\phi(r) = \frac{-Gm}{r}.$ This describes the effect of the gravitational field of an object that has a mass of **m** at a potential distance **r**. These descriptions of gravity are vital for contributing to our understanding of physics and helps us get that much closer to understanding our very complicated universe. The uses for a description of gravity of course cannot be overstated, as every satellite is reliant on an understanding of gravity to not come crashing through the atmosphere or to float away into space. We can also calculate the mass of celestial bodies with some algebra.

Electrostatics

Poisson’s equation is one of the pivotal parts of Electrostatics, where we would solve the equation to find electric potential from a given charge distribution. In layman’s terms, we can use Poisson’s Equation to describe the static electricity of an object. While it may not seem it, static electricity comes up surprisingly often in our daily lives just in more subtle ways than we may perhaps realize. Lightning can be described as a large scale electrostatic event. All electronic devices are incredibly vulnerable to static electricity, which is why people ground themselves when they construct computers. For people who have wireless charging phones, you have electrostatics to thank for such a development for that convenience.

$$\nabla^2 \varphi = -\frac{\rho}{\varepsilon}$$

Works Cited

https://en.wikipedia.org/wiki/Poisson%27s_equation

https://en.wikipedia.org/wiki/Electrostatics#'Static'_electricity

https://en.wikipedia.org/wiki/Newton%27s_law_of_universal_gravitation#Gravitational_field

https://en.wikipedia.org/wiki/Sim%C3%A9on_Denis_Poisson